The Economics of Crowdfunding

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Abstract

An entrepreneur finances her project via crowdfunding. She chooses a funding mechanism (fixed or flexible), a price, and a funding goal. Under fixed funding money is refunded if the goal is not met; under flexible funding there is no refund. Backers observe signals about project value and decide whether to contribute or postpone purchase to the retail stage. Using the linkage principle, we show that the optimal campaign uses fixed funding. Furthermore, we show that an entrepreneur who is not financially-constrained can approximately attract full surplus using fixed funding. Therefore, crowdfunding is attractive to both small and large entrepreneurs.

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1 Introduction

Crowdfunding has become a popular means for small entrepreneurs to finance their projects, typically through online platforms, where the entrepreneur asks a large number of internet users to back up her project with money. Despite being a relatively new global phenomenon, it has been growing exponentially. Its global market size, which tops over 30 billion dollars in 2015, is already larger than the market size for angel funds and will soon be larger than that of venture capital. Moreover, the U.S. government has been deregulating their equity crowdfunding market, allowing non-accredited investors to join the game. It is thus of great interest and importance to understand how and why crowdfunding works and to provide a rationale for deregulation.

This paper proposes a common value model of crowdfunding that explains the success of crowdfunding despite the lack of regulation and potential moral hazard problems. We consider a risk-neutral entrepreneur who would like to crowdfund her project with a fixed cost from a continuum of risk-neutral backers. The project has a common value unknown to the entrepreneur but the backers are partially informed. Using a linkage principle argument, we give a revenue ranking result between the two most popular formats of crowdfunding. Furthermore, we show that an entrepreneur who is not financially-constrained can approximately attract full surplus using crowdfunding.

In a crowdfunding campaign, the entrepreneur posts the description of her project to a third party funding platform, chooses a funding mechanism, sets a funding goal, sets a price each backer pays, and sets the reward each backer gets.\footnote{This posted price mechanism is used in, for example, the two largest crowdfunding platforms Kickstarter and Indiegogo. The entrepreneur usually offers different reward levels for different prices, for example to get an album one needs to pay $20, to get an autographed album with poster one needs to pay $50. However, more often than not, the most popular option is the lowest price that can get the backers a unit of the good. We simplify this aspect so we do not consider price discrimination. For a treatment, see Ellman & Hurkens (2016).} The reward is typically a unit of the good her project is aimed to produce, or it can be a share of the company.

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as well. Our model admits both interpretations. In practice, there are two very popular choices of posted-price funding mechanism\(^2\): fixed or flexible. Under fixed funding, the money is refunded if the goal of the campaign is not met, while under flexible funding there is no refund, whatsoever.\(^3\) The backers receive conditional i.i.d. signals about the value of the project and decide whether to contribute or postpone their purchase to the retail stage. Figure 1 provides an example of a crowdfunding campaign.\(^4\)

![Ember - Temperature Adjustable Mug](https://www.indiegogo.com/projects/ember-temperature-adjustable-mug)

Figure 1: The entrepreneur in this Indiegogo project wants to raise $50,000 to develop a temperature preserving mug. Each backer needs to contribute at least $109 in order to receive a mug after it is produced. The entrepreneur adopts flexible funding, so the entrepreneur will receive all the funds even if the campaign fails to reach its funding goal.

Our first result (Theorem 1) shows that fixed funding generates more profit than

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\(^2\)In equity funding or debt crowdfunding more complicated mechanisms are sometimes used. For example, payment can be a function of the bids.

\(^3\)The largest reward-based crowdfunding platform Kickstarter only allows fixed funding, while the campaigns from its biggest competitor, Indiegogo, are predominantly flexible funding.

\(^4\)Link: [https://www.indiegogo.com/projects/ember-temperature-adjustable-mug/](https://www.indiegogo.com/projects/ember-temperature-adjustable-mug/)
flexible funding. At first, this may seem obvious: people are more willing to pay more if they are refunded when the project does not go ahead. But this logic is incorrect. For example, suppose a project has 50% chance of being funded. Then backers are indifferent between paying $50 under fixed funding and $25 under flexible funding, and the expected revenue to the entrepreneur is the same in either case. When backers have common value but heterogeneous beliefs about the value—which is appropriate when quality is unknown— the equivalence breaks down and fixed funding becomes preferable to the entrepreneur. Intuitively, suppose the marginal backer thinks that the project will be funded with probability 50% so he is indifferent between paying $50 under fixed funding and $25 under flexible funding. A high signal backer thinks that the project will be funded with more than 50% probability, so he has higher expected payment under fixed funding than flexible funding.

This logic is akin to the linkage principle that ranks the revenue of the all-pay auction over that of the first-price auction in an interdependent value setting. Flexible funding works like the all-pay auction: in the all-pay auction (flexible campaign), a bidder (backer) pays what he bids regardless of his signal or the outcome. Fixed funding works like the first-price auction in a reversed way. In the first-price auction (fixed campaign) a bidder (backer) with a higher signal has a lower (higher) chance to win since other bidders are more competitive (other backers are more enthusiastic), therefore less (more) expected payment. Since the expected payment of the fixed campaign is more positively correlated with the signals than that of the flexible campaign, the linkage principle implies that fixed

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5 The market value of the mug may depend on the price of the complements, say tea and coffee, or the existence of competing substitutes, like a much cheaper temperature preserving mug. In this sense the crowd can be more informed than the entrepreneur. Traditionally the entrepreneur will run surveys to focus groups to extract the information.

6 The linkage principle first appeared in Milgrom & Weber (1982) as the incentives to commit to reveal information and is stated in a more unified form in Proposition 7.2 of Krishna (2010) as a revenue ranking result according to how responsive (in a positive way) a bidder’s equilibrium expected payment to his signal is.
funding has a higher revenue.

This result also explains the popular conception that fixed funding projects perform better than flexible projects, which is supported by comparisons of the two of the largest crowdfunding platforms, Kickstarter and Indiegogo (Cumming et al. (2014)), and by within comparison of projects in Indiegogo.\(^7\)

The second result (Theorem 2) studies the optimal fixed funding campaign and shows that the entrepreneur can approximately extract full surplus is she is financially unconstrained. This explains why an entrepreneur wants to use crowdfunding regardless of how much fund she needs. Crowdfunding is costly because the backers need to be given information rents. Therefore the entrepreneur will not ask for more than needed if she has other less-costly sources of money. On the other hand, crowdfunding still helps the entrepreneur to learn the market value of her idea and to condition the building decision on the market value. Indeed, a growing number of enterprises, including Sony and GE, have been using crowdfunding (in a smaller scale compared to their retail markets) as a means to gain customer feedbacks of their innovations.

**Related Literature**

Crowdfunding can be considered as a model in which a monopolist makes a production decision together with the financing of it through advance-purchase contracts. Models with private value are first studied by Cornelli (1996), in a setting where the monopolist can commit to build the project. Recent papers, such as Strausz (2017) and Chemla & Tinn (2016), take moral hazard, which is a crucial issue of crowdfunding, into account. While these papers are in a “private value” environment, the private values are either zero or one. Therefore, their models focus on how crowdfunding sort out consumers interested in the project from those not interested at all. For those interested, they have the same


value and have no value uncertainty when they contribute to a campaign. On the other hand, our model assumes that the consumers that browse the particular crowdfunding campaign are already interested, but they are informationally heterogeneous about the project’s underlying common value.

Models with private values have also been considered to study other issues than moral hazard. Ellman & Hurkens (2016) consider price discrimination. An earlier paper by Belleflamme et al. (2014) compare pre-order and profit-sharing when backers get additional community utilities if they contribute. Grüner & Siemroth (2018) focuses on the efficiency of equity crowdfunding where the market for crowdfunded good is competitive and backers are credit-constrained.

There are few papers regarding common values other than this paper. A relevant one is Hakenes & Schlegel (2014), who study a finite-agent common value crowdfunding model with endogenous information acquisition. In their paper, a firm can be good or bad and the backers need to pay a cost to get a noisy signal about it. They analyze the mixed strategy equilibrium, in which a lower funding threshold translates to higher equilibrium probability for the backers to get informed. They find that in an optimal crowdfunding campaign, all backers pay to get informed and contribution is made if and only if the signal is good. In our common value model, a like-wise result obtains: a lower funding threshold makes the backers more conservative about contribution.

Assuming that the production decision has been made, Nocke et al. (2011) studies a monopolist screening consumers with private but noisy values using pre-order discounts so that agents with high signals purchase in advance and agents with lower signals postpone the purchase until the value realizes in the retail stage.

The idea of fixed funding, where money is refunded if the total amount fails to reach a threshold, is simply the provision-point mechanism. This mechanism has been widely studied in the private provision of public goods due to its efficiency in complete informa-

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8A correction to some of the results is by Sahm et al. (2014)
9Cornelli (1996) finds that the optimal mechanism in her environment is also akin to a provision-point
tion environments (Bagnoli & Lipman (1989)). The efficiency of this mechanism has also been experimentally demonstrated (Palfrey & Rosenthal (1988), Rondeau et al. (1999), Rondeau et al. (2005), Spencer et al. (2009)).

This paper also contribute to models of information aggregation and allocation of an excludable good with common value (Milgrom & Weber (1982), Pesendorfer & Swinkels (2000)). In particular, we propose a novel application of the linkage principle in a crowdfunding environment. However, their papers focuses on conflicts of information aggregation and allocative efficiency, (Grossman & Stiglitz (1980)), while in our model information aggregation is automatically granted, and monopoly power only distorts allocative efficiency.

Finally, it is also of interest to focus on crowdfunding platforms as intermediaries of a two-sided market. For an overview of related economic issues, see Belleflamme et al. (2015).

The paper is organized as follows: Section 2 outlines the crowdfunding model. Section 3 presents the main results. Section 4 concludes. Proofs can be found in the appendix.

# 2 The Model

This section outlines the model.

**Players** An entrepreneur (she) tries to fund a project through crowdfunding. A continuum of potential backers (he) decide whether to contribute to a project in return for a unit of the good the project aims to produce.

**Project** A project has a fixed cost $k$ and zero marginal cost. A project generates common value $v$ to the backers. $v$ is unknown to both sides of the market with common prior $f(\cdot)$ on $[0, 1]$. Backers privately receive conditional i.i.d. signals, $s$, about $v$ according to the conditional density $g(s|v)$ with cdf $G(s|v)$. We assume that the entrepreneur can

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mechanism. Specifically, in her model it is the sum of some increasing function of payments instead of the sum of payments that determine when a project is built.
commit to build the project whenever it is funded, which is the case when the value of
her reputation is worth more than what she can gain by not fulfilling the promises. In
this case, it is inconsequential whether the backers know $k$.\textsuperscript{10}

The following assumptions are made throughout the paper.

A1 \( \{g(s|v)\} \) satisfies strict monotone likelihood ratio property (MLRP).

A2 \( g(s|v) \) is continuous on \([0, 1] \times [0, 1]\) and \( g(s|v) > 0 \) for all \( s \in [0, 1], v \in [0, 1] \),
\( f(v) > 0 \) for all \( v \in [0, 1] \).

**Efficiency Benchmark** Since we assume the backers eventually buy the product,
only the building decision affects efficiency. Social optimum obtains if the project is built
whenever its value is larger than its cost:

\[
SW = \int_k^1 (v - k)f(v)dv
\]

**Crowdfunding Campaign** The entrepreneur has an initial amount of asset $a$, where
\( 0 \leq a \leq k \). When \( a = k \), the entrepreneur is not financially constrained. A natu-
ral constraint is \( a = 0 \), which means that the entrepreneur’s only source of funding is
crowdfunding.

A crowdfunding campaign is a tuple \((F, T, p)\), where

\[
F \in \{Fix, Flex\}
\]
denotes the funding mechanism, \( T \) represents the commitment to build the project if and
only if the seller gathers at least \( T \) dollars, and \( p \) is the pledge price each buyer has to
pay if he is to contribute.

Under fixed funding, the entrepreneur gets money if and only if she raises at least
\( T \) dollars. Under flexible funding, the entrepreneur always gets the money even if she
\textsuperscript{10}When the entrepreneur has no commitment power and $k$ is common knowledge, the same results
continue to hold. Intuitively, when moral hazard is present, a fixed funding campaign on the third-party
crowdfunding platform is a commitment device: the project is designed to be funded if and only if the
value is high, which is when the entrepreneur has incentives to build the project.
raises less than her goal. In both mechanisms, the entrepreneur commits to implement
the project if and only if at least $T$ dollars are collected.

When the entrepreneur commits to build the project whenever it’s funded, she has no
choice but to choose $T \geq k - a$ and $p \geq T$. The first inequality ensures that whenever
the project is funded she will have enough money (alongside with her asset $a$) to cover
the project’s cost, and the second inequality ensures that the campaign has a chance
to succeed if it attracts enough backers (from a continuum of backers with mass one).
Formally, the entrepreneur chooses $(T, p)$ from the set

$$C_a = \{(T, p) : k - a \leq T \leq p \leq 1\}.$$

**Actions** The entrepreneur chooses a crowdfunding campaign $(F, T, p)$ where $(T, p) \in
C_a$. After observing $(F, T, p)$ and signal $s$, the backers simultaneously choose whether to
contribute $p$ dollars or wait to purchase at the retail stage. An action profile for backers is
a measurable function $\sigma : [0, 1] \rightarrow [0, 1]$, where $\sigma(s)$ denotes the probability to contribute
for a backer with signal $s$. We assume that the value of the project is realized at the
retail stage and that the retail price equals the value.

Given a price $p$ and an action profile $\sigma$, the money that will be contributed at each
state $v$ is then

$$X_0^\sigma(v) = \int_0^1 p\sigma(s)g(s|v)ds.$$

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11 The actual crowdfunding mechanisms are mostly dynamic, where the backers can observe the current
total contribution. Theoretically, backers can strategically postpone their contribution until they are sure
the project is funded. However, empirical contribution dynamics are usually U-shaped with respect to
time, so only a subset of backers will strategically postpone purchase. Their behavior, however, is not
relevant to determine whether a project is funded.

12 This assumption enables the entrepreneur to extract full surplus after the product is made. However,
without resorting to crowdfunding, the entrepreneur could have built the project when the value is low
and thus loses money. Hence, even if she has sufficient fund, the entrepreneur will still use crowdfunding.

13 This assumption is not needed for our results but it simplifies the exposition.
Entrepreneur sets \((F,T,p)\)

Money collected or refunded

Entrepreneur builds the project iff at least \(T\) is raised.

Backers observe signal \(s\)

Backers decide whether to contribute \(p\)

\(v\) realizes at retail stage.

Figure 2: Timeline of the crowdfunding campaign model

For each state, the retail stage revenue the entrepreneur gets (if the project is built) is

\[
X_1^\sigma(v) = \int_0^1 v(1 - \sigma(s))g(s|v)ds.
\]

The project is funded at state \(v\) if \(X_0^\sigma(v) \geq T\). For an action profile \(\sigma\),

\[
B^\sigma = \{v : X_0^\sigma(v) \geq T\}
\]

is the set of states under which the project is funded.

### 2.1 Payoffs and Profits in a Fixed Funding Campaign

**Backers’ Payoff** Given \((Fix,T,p)\) and an action profile \(\sigma\), the expected utility of a backer with signal \(s\) to contribute is

\[
U^{Fix}(s; \sigma) = \int_{B^\sigma} (v - p)\beta(v|s)dv
\]

where

\[
\beta(v|s) = \frac{g(s|v)f(v)}{\int_0^1 g(s|v)f(v)dv}
\]

is the backer’s posterior about the project value if he observes signal \(s\). Since this is a fixed funding campaign, there are no transactions if the project is not funded. Therefore the backers have non-zero utilities only when \(v \in B^\sigma\). Once a backer receives a signal \(s\), he has a posterior about the distribution of \(v\) given \(s\), and make contribution decisions based on the posterior.\(^{14}\)

\(^{14}\)Note that this differs from Feddersen & Pesendorfer (1997) in that the backers do not condition their posteriors on the event that they are pivotal. This is because, even in a finite agent model, the backers’
Entrepreneur’s Profit The entrepreneur’s profit under \((Fix, T, p)\) and action profile \(\sigma\) is the sum of crowdfunded money, retail stage revenue minus the project cost

\[
\Pi^{Fix}(T, p; \sigma) = \int_{B^\sigma} (X_0^\sigma(v) + X_1^\sigma(v) - k)f(v)dv
\]  

(1)

2.2 Payoffs and Equilibria in a Flexible Funding Campaign

Backers’ Payoff Given \((Flex, T, p)\) and an action profile \(\sigma\), the expected utility of a backer with signal \(s\) to contribute is

\[
U^{Flex}(s; \sigma) = \int_{B^s} v\beta(v|s)dv - p,
\]

Note that in a flexible funding campaign, backers always pay but receive a payoff only when the project is funded, hence the expected utility differs from that of fixed funding.

Entrepreneur’s Profit The entrepreneur’s profit under \((Flex, T, p)\) and action profile \(\sigma\) is

\[
\Pi^{Flex}(T, p; \sigma) = \int_{0}^{1} X_0^\sigma(v)f(v)dv + \int_{B^\sigma} (X_1^\sigma(v) - k)f(v)dv
\]  

(2)

Remark 2.1. Because of the common value, full surplus can be extracted (Crémer & McLean (1988)) by a direct incentive compatible mechanism: simply ask each backer to announce his signal, and the good is produced and allocated with a price equal to the value uniquely pinned down by the distribution of signals, whenever the value is larger than the cost. However, such a mechanism is hardly feasible in practice. We focus on posted-price mechanisms that resemble the ones used in practice and we show in Theorem 2 that fixed funding approximately extracts full surplus when the entrepreneur is not credit-constrained.

utility of contributing or waiting differs whenever the project is funded (fixed funding). On the other hand, in a voting model an action leads to a difference in utility only when the voter is pivotal.
2.3 The Equilibrium Notion

**Equilibrium** An action profile, \( \sigma \), is a Bayes Nash equilibrium under \((F,T,p)\) if for all \( s \in [0,1] \),

\[
U^F(s; \sigma) > 0 \Rightarrow \sigma(s) = 1 \\
U^F(s; \sigma) < 0 \Rightarrow \sigma(s) = 0
\]

For any campaign, zero contribution is always an equilibrium. That is, \( \sigma(s) = 0 \) for all \( s \). However, we are interested in equilibria under which the project is funded with positive probability. We call an equilibrium funded if \( \int_{B^*} f(v) dv > 0 \). Furthermore, an action profile \( \sigma \) is a cutoff strategy if there exists \( s^* \in [0,1] \) such that \( \sigma(s) = 0 \) whenever \( s < s^* \) and \( \sigma(s) = 1 \) whenever \( s > s^* \).

Our first observation is that any funded equilibrium of a fixed funding campaign is a cutoff strategy, and that for any funded equilibrium \( \sigma \) of a flexible funding campaign, there is a funded equilibrium in cutoff strategies \( \tilde{\sigma} \) such that \( \tilde{\sigma}(s) \geq \sigma(s) \) for all \( s \in [0,1] \).

**Lemma 2.1.** 1. Let \( \sigma \) be a funded equilibrium of the campaign \((\text{Fix}, T, p)\), then \( \sigma \) is a cutoff strategy with cutoff \( s^* \in [0,1) \).

2. Let \( \sigma \) be a funded equilibrium of the campaign \((\text{Flex}, T, p)\), then there is a funded equilibrium in cutoff strategies \( \tilde{\sigma} \) such that \( \tilde{\sigma}(s) \geq \sigma(s) \) for all \( s \in [0,1] \).

The intuition for fixed funding is as follows: a backer’s expected utility of funding a project is the conditional probability a project is funded times the expectation of \( v - p \) conditional on being funded. For an arbitrary \( \sigma \), if a backer with signal \( s \) thinks that \( v - p \) is positive conditional on being funded, then a backer with higher signal would also find it positive and therefore would contribute.

The previous intuition fails for flexible funding because, if \( \sigma \) is such that only people with intermediate signals contribute, higher signal may imply that the project is less likely to be funded while payment \( p \) still has to be made regardless of whether the project is
funded. Hence higher signals may not translate to higher incentives to contribute. On the other hand, potential backers with high signals would be willing to contribute if they think others are contributing when they receive high signals.

In light of Lemma 2.1, we will focus on cutoff equilibria throughout the paper.\textsuperscript{15} Under a funded equilibrium with cutoff \(s^*\), the project is funded whenever the underlying value \(v\) is above some threshold \(v^*\) because the crowdfunded money \(X^\sigma_0(v) = p(1 - G(s^*|v))\) is increasing in \(v\). We refer to \((s^*, v^*)\) as the equilibrium cutoffs.

### 3 The Results

In this section we show that fixed funding is revenue-superior to flexible funding and that it is able to approximately extract full surplus when the entrepreneur is not financially constrained.

We first show existence and uniqueness of funded equilibrium in fixed funding. Note that equilibrium existence is independent of the prior and the signal structure other than that they satisfy assumption A1 and A2.\textsuperscript{16}

**Proposition 3.1.** Given \((\text{Fix}, T, p)\) with \((T, p) \in C_a\). Suppose

\[
T < p < 1
\]

then an unique funded equilibrium exists. Conversely, if a funded equilibrium exists then \(T \leq p < 1\).

The intuition is that if all other backers use a high cutoff \(s^*\), then the project will be funded only when the value is high, and one will be incentivized to contribute even if he

\textsuperscript{15}We do not have a proof that the profit-maximizing equilibrium of flexible funding is in cutoff strategies. However, equilibria involving high signal backers not contributing seem unlikely to be profit-maximizing.

\textsuperscript{16}The condition under which a cutoff-strategy equilibrium in the flexible campaign \((\text{Flex}, T, p)\) would exist depends on the signal structure and is relegated to the appendix (see Proposition B.1).
observes a low signal. The best response function (the continuity of which is shown in the appendix) is thus downward sloping and crosses the 45-degree line once.

Thanks to Proposition 3.1, we can now unambiguously write $\Pi^{Fix}(T, p)$ as the profit of the fixed funding campaign when $(T, p) \in C_a$ (the profit is zero if $T = p$ and a funded equilibrium does not exist). We are now ready to state our first main result.

**Theorem 1.** For any $0 \leq a \leq k$ and for any funded equilibrium in cutoff strategy, $\sigma$, under $(Flex, T, p)$, where $(T, p) \in C_a$, there exists $(T', p') \in C_a$ with $T' < p' < 1$ such that

$$\Pi^{Fix}(T', p') > \Pi^{Flex}(T, p; \sigma).$$

The intuition is as follows. Suppose that under campaign $(Flex, T, p)$ the marginal backer, $s^*$, thinks the project is funded with probability 0.5, and a backer with a higher signal $s > s^*$ thinks the project will be funded with probability 0.75. Their expected utilities for contributing are, respectively,

$$U^{Flex}(s^*) = 0.5\mathbb{E}[value|s^*,\text{funded}] - p = 0$$
$$U^{Flex}(s) = 0.75\mathbb{E}[value|s,\text{funded}] - p > 0$$

Suppose the entrepreneur now switches to fixed funding but doubles the price $p$ to $2p$, and adjusts the corresponding $T$ properly so that $v^*$ stays unchanged.\(^{17}\) We then have

$$U^{Fix}(s^*) = 0.5\mathbb{E}[value|s^*,\text{funded}] - 0.5\mathbb{E}[2p|s^*,\text{funded}] = 0$$
$$U^{Fix}(s) = 0.75\mathbb{E}[value|s,\text{funded}] - 0.75\mathbb{E}[2p|s,\text{funded}] < U^{Flex}(s)$$

This adjustment makes the marginal backer as happy as before. Moreover, it extracts more surplus from high signal backers, so it leads to a higher profit.

To see the linkage principle more clearly, note that if $(Fix, T', p')$ and $(Flex, T, p)$ implement the same cutoff $(s^*, v^*)$\(^{18}\), then by the indifference of the marginal backer,

$$p = (1 - B(v^*|s^*))p'$$

\(^{17}\)Technically, the entrepreneur sets $T' = 2p(1 - G(s^*|v^*)) = 2T$.

\(^{18}\)Lemma A.3. shows that such $(T', p')$ exists
where \( B(v^*|s) = \int_0^{v^*} \beta(w|s)dw \) is the probability assessed by a backer with signal \( s \) of getting a refund. The payoffs for a backer with signal \( s \) under these two campaigns are therefore

\[
U^{\text{Fix}}(s) = \int_{v^*}^1 v\beta(v|s)dv - \frac{1 - B(v^*|s)}{1 - B(v^*|s^*)}p
\]

\[
U^{\text{Flex}}(s) = \int_{v^*}^1 v\beta(v|s)dv - p
\]

By MLRP, the expected payment for type \( s \) under fixed funding is strictly higher than that under flexible funding. Since allocations are the same, the profit under fixed funding is higher.

Because the allocations before and after the adjustment are the same, the increase in expected profit comes entirely from crowdfunding. The result is thus independent of the choice of retail price (which we have assumed to be the realized value), as long as the retail price is non-decreasing in \( v \) and that it is higher than \( p \) with positive probability. For example, one can choose the retail price to be 1, which amounts to shutting down the retail market. Nevertheless, when there are moral hazards, profitability in the retail market provides incentives for the entrepreneur to keep the promise.

**Remark 3.1** (Robustness to Variations of Profit Function). *Our method of proof is not based on the comparison of profits under optimal pricing. Instead, it is based on a feasibility argument. For fixed funding the entrepreneur can achieve the same allocation (decision to build \( v^* \) and who contributes \( s^* \)) with less consumer surplus and a higher funding goal. Hence the revenue ranking is robust to various modifications of the profit function.*

**Remark 3.2.** *Theorem 1 also encompasses the case where \( T = 0 \) (and thus \( v^* = 0 \)) in a flexible funding campaign. In such a case, the dominating fixed campaign \((\text{Fix}, T', p')\) will be chosen in a way that a higher cutoff \( v^* > 0 \) is implemented. By doing so the entrepreneur can both increase social surplus and reduce consumer surplus. Indiegogo states that entrepreneur should only choose flexible funding "if any amount of money will

\[19\text{See the discussion after Theorem 2 for examples of the modifications.} \]
help you reach your campaign objective and you will still be able to fulfill your perks”,
which amounts to choosing flexible funding only when the entrepreneur has enough assets
to begin with ($a = k$). The project will be built even if the value is very low, which is bad
for profits.

Having shown that fixed funding generates more expected profit, we now take a closer
look of fixed funding. The entrepreneur’s problem is given by

$$\max_{(T,p) \in C_a} \Pi^{Fix}(T,p),$$

where the profit for any choice of $(T,p) \in C_a := \{(T,p) : k - a \leq T \leq p \leq 1\}$ is defined to
be the one given by the unique funded equilibrium when there exists one, and zero when
it does not exist.

Our second main result studies optimal fixed funding and shows how an entrepreneur
would use crowdfunding to her best interest. In particular, an entrepreneur who is not
credit-constrained can approximately extract full surplus.

**Theorem 2.**

(a) (Optimal Campaign) For every $0 \leq a < k$, the entrepreneur’s problem has a solution.
Moreover, at the optimum, $T = k - a < p < 1$, $v^* \geq k$.

(b) (Approximate Full Surplus Extraction) For $a = k$, there exists a sequence of $\{(T_n,p_n)\} \subset C_k$ such that funded equilibrium exists for each $n$ and that

$$\lim_{n \to \infty} \Pi^{Fix}(T_n,p_n) = \int_k^1 (v - k)f(v)dv. \quad (3)$$

Moreover, $\lim_{n \to \infty} T_n = 0$, $\lim_{n \to \infty} (s^*_n, v^*_n) = (1,k)$, where $s^*, v^*$ are corresponding
equilibrium cutoffs.

When $a < k$, the entrepreneur chooses $T = k - a$. This is because asking for money
from the crowd incurs information rents, so the entrepreneur will never ask for more than
what is needed.\textsuperscript{20} In the optimal campaign, we have the typical result that the monopolist supplies less than what is socially efficient: \( v^* \geq k. \textsuperscript{21} \\

The case \( a = k \) means that the entrepreneur is not financially constrained. There is then a simple mechanism to approximately extract full surplus. The entrepreneur simply runs an epsilon crowdfunding campaign designed to be funded exactly when \( v \geq k \), and then the entrepreneur can safely invest her asset to the project and extract the surplus.\textsuperscript{22}

Note that, however, if sequential fund-raising is allowed (which is not considered in the paper), then credit constraint \( a < k \) does not prevent the entrepreneur from full surplus extraction: the entrepreneur simply runs an epsilon crowdfunding campaign designed to be funded exactly when \( v \geq k \). After the campaign is funded, the rest of the consumers will be willing to pay \( v \geq k \) in advance, or the entrepreneur can convince the market to lend her \( k \) dollars at the risk-free rate.\textsuperscript{23}

There are plenty of reasons why full surplus extraction is not attainable even if the entrepreneur is not credit-constrained. For one thing, it may be costly to use the outside assets. For another, the number of backers may have an advertisement effect such that the

\textsuperscript{20}This logic also applies to the flexible funding campaign.
\textsuperscript{21}Suppose \( p \) is chosen such that \( v^* = k \). There is no first order effect to social surplus when \( p \) is increased. However, there may be two opposite first order effects to consumer surplus: As \( p \) increases, consumer surplus decreases because one has to pay more. On the other hand, \( v^* \) may also increase. This increases consumer surplus since \( v^* < p \) in equilibrium, so it’s less likely for consumers to get a product with value lower than what they are paying. Which effect is stronger depends on the parameters of the model.
\textsuperscript{22}In 2016, Indiegogo launched its Enterprise initiative, which encourage corporate users to use crowdfunding as a means to validate their market research (https://enterprise.indiegogo.com/). This leads to quite a few number of successful projects. As a successful example, Bose, a leading company in the audio industry, attracted about three thousand backers to its Sleepbuds project on Indiegogo. The backers paid about $160 for a pair. The Sleepbuds are now sold on Amazon.com for $249 a pair.
\textsuperscript{23}It is not uncommon for the entrepreneur to first prove that her product is profitable by selling to a smaller audience and then acquiring the larger portion of funding needed for the project from venture capitalists. A famous example is Oculus, which is a virtual reality headgear, that raised $2.5M through nearly 10,000 backers on Kickstarter and was acquired by Facebook subsequently for $2B.
number of retail consumers is increasing in the number of backers. It is also reasonable to assume that the backers have a positive rate of not going to the retail stage if they choose not to contribute in the crowdfunding stage. Their attention can shift elsewhere, as there may be substitutes for the product. For the entrepreneur, these customers are forever lost if they are not attracted to contribute in the first place. All of these features can be incorporated into our model without affecting Theorem 1.

4 Conclusion

This paper models crowdfunding as an entrepreneur posting a price to partially informed backers. The backers’ incentive to contribute to the campaign is driven by the expectation of paying a lower price than the retail price. Fixed funding reinforces this incentive by refunding the money when the value of the good is low, and therefore it has a higher chance of getting funded and also raises more funds.

Our model thus explains the success of crowdfunding and in particular the popularity of fixed funding. Measures to deregulate the equity crowdfunding market, such as the JOBS act, thus seem to be a right step.24

Despite that fixed funding is considered to be better than flexible funding both empirically (Cumming et al. (2014)) and theoretically, flexible funding is still widely adopted in donation based projects, where the backers contribute for altruistic reasons and demand no return, and in some Indiegogo projects as well. The Indiegogo projects that use flexible funding are often those that have been partially completed and thus the decision to build has been made and does not depend on the outcome of the crowdfunding campaigns, or those that are scalable. These are distinct types of projects than the ones modeled in this paper.

24In equity funding, however, more complex pricing schemes are used. At the very least, the underlying good becomes divisible since the entrepreneur can sell shares to the investors. Profit-sharing may create its own kind of moral hazard problems.
There are several interesting directions one can further investigate. For instance, one can extend the model to scalable projects where the entrepreneur may need to choose an investment policy. Another direction is to consider the dynamics of crowdfunding and study how information is aggregated and how the current backers learn through observing previous backers, which relates to a vast learning/herding literature pioneered by Bikhchandani et al. (1998). It is also of interest to more extensively study equity funding and see how it compares to methods such as going through an IPO.

References


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25When projects are scalable, backers’ contribution decisions display complementarity: the lower the cutoff used by others, the more the entrepreneur can invest to the project, and the final product will be better. This potentially leads to multiple equilibria in cutoff strategies.


A Proofs

First we give a property of MLRP that will be used subsequently. Let \( \{g(x|y)\} \) be probability densities satisfying strict monotone likelihood ratio property. For a measurable set \( A \subset \mathbb{R} \), let \( G(A|y) = \int_0^1 1_A(x)g(x|y)dx \) denote the conditional probability of event \( A \) under density \( g(x|y) \). Let

\[
g_A(x|y) = \begin{cases} 
g(x|y)/G(A|y) & x \in A \\ 0 & x \neq A \end{cases}
\]

be the conditional density conditioning on \( A \), and denote the conditional distribution by \( G_A(x|y) \).

**Lemma A.1.** Let \( A \subset [0,1] \) be measurable and that \( G(A|y), G(A|y') > 0 \). For \( y < y' \) and a non-constant increasing function \( h(\cdot) \),

\[
\int_A h(x)g_A(x|y)dx < \int_A h(x)g_A(x|y')dx.
\]

**Proof.** MLRP implies for \( y' > y \), \( g_A(x|y')/g_A(x|y) \) is increasing w.r.t. \( x \in A \) and since \( g_A(x|y), g_A(x|y') \) are probability densities, there exists some \( x^* \in A \) such that for all \( 0 \leq x_1 \leq x^* \leq x_2 \),

\[
\frac{g_A(x_1|y')}{g_A(x_1|y)} \leq 1 \leq \frac{g_A(x_2|y')}{g_A(x_2|y)},
\]

with strict inequality when \( x_1 < x^* < x_2 \). For \( x \geq x^* \), we then have

\[
1 - G_A(x|y) = \int_x^1 g_A(s|y)ds \leq \int_x^1 g_A(s|y)\frac{g_A(s|y')}{g_A(s|y)}ds = \int_x^1 g_A(s|y')ds = 1 - G_A(x|y').
\]

For \( x \leq x^* \) we have

\[
G_A(x|y) = \int_0^x g_A(s|y)ds \geq \int_0^x g_A(s|y)\frac{g_A(s|y')}{g_A(s|y)}ds = \int_0^x g_A(s|y')ds = G_A(x|y').
\]

Hence, \( G_A(x|y) \) satisfies FOSD, which implies that the expected value of an increasing function under \( G_A(x|y') \) dominates that under \( G_A(x|y) \). Since \( G_A(x|y) \neq G_A(x|y') \) for \( y \neq y' \) the inequality is strict.

\( \square \)
A.1 Proofs for Section 2

Proof of Lemma 2.1. Let $\sigma$ be a funded equilibrium of the campaign $(\text{Fix}, T, p)$. Then there exists $s < 1$ such that $U^{\text{Fix}}(s; \sigma) \geq 0$. Suppose there exists no $s^* \in [0, 1]$ such that $U^{\text{Fix}}(s^*; \sigma) = 0$, then by the continuity of $U^{\text{Fix}}(s)$, it has to be $U^{\text{Fix}}(s) > 0$ and then $\sigma$ is a strategy with cutoff zero. Suppose $U^{\text{Fix}}(s^*; \sigma) = 0$ for some $s^*$. That is,

$$U^{\text{Fix}}(s^*; \sigma) = \int_{B^\sigma} (v - p) \beta(v|s^*)dv = \mathbb{P}(B^\sigma|s^*) \int_{B^\sigma} (v - p) \frac{\beta(v|s^*)}{\mathbb{P}(B^\sigma|s^*)} dv = 0,$$

Since $\{g(s|v)\}$ satisfies strict MLRP, the set of posteriors $\{\beta(v|s)\}$ also satisfies strict MLRP. Applying Lemma A.1. to the integral $\int_{B^\sigma} (v - p) \beta(v|s)dv$, we obtain that $U^{\text{Fix}}(s; \sigma) > 0$ when $s > s^*$ and $U^{\text{Fix}}(s; \sigma) < 0$ when $s < s^*$. Hence the equilibrium $\sigma$ under fixed funding is characterized by the cutoff $s^*$.

Let us turn to flexible funding. Let $\sigma$ be a funded equilibrium of the campaign $(\text{Flex}, T, p)$. Let $s_1 = \inf\{s : U^{\text{Flex}}(s, \sigma) \geq 0\}$ be the lowest signal a backer with which would be willing to contribute. Let

$$\sigma_1(s) = \begin{cases} 
0 & s < s_1 \\
1 & s \geq s_1
\end{cases}$$

Then, since $B^\sigma \subset B^{\sigma_1}$, we have $U^{\text{Flex}}(s, \sigma_1) = \int_{B^{\sigma_1}} v \beta(v|s)dv - p \geq 0$ for all $s \geq s_1$. Inductively, define $s_n = \inf\{s : U^{\text{Flex}}(s; \sigma_{n-1}) \geq 0\}$. Since $B^{\sigma_{n-1}} \subset B^{\sigma_n}$, $s_n$ is a decreasing sequence, which converges to some $s^*$. Finally, define

$$\tilde{\sigma}(s) = \begin{cases} 
0 & s < s^* \\
1 & s \geq s^*
\end{cases}$$

By construction, $\tilde{\sigma}$ is a funded equilibrium in cutoff strategies with $\tilde{\sigma}(s) \geq \sigma(s)$ for all $s$.

A.2 Proofs for Section 3

Proof of Proposition 3.1. By Lemma 3.1 we only need to focus on cutoff strategies.
Let $\sigma_s$ be the cutoff strategy with cutoff $s$. For each $s \in [0, 1]$, let

$$v^f(s) = \min \{ v : p(1 - G(s|v)) \geq T \}$$

be the minimum value $v$ above which the project is funded, given that backers use $\sigma_s$. Note that since $p > T$, $v^f(\cdot)$ is continuous. Also, $v^f(\cdot)$ is increasing.

Define $\Phi : [0, 1] \rightarrow [0, 1]$ as

$$\Phi(s) = \min \left\{ s' : \int_{v^f(s)}^1 (v - p)\beta(v|s')dv \geq 0 \right\}.$$ 

Note that whenever $v^f(s) < 1$, by Lemma A.1 the conditional expected utility $\int_{v^f(s)}^1 (v - p)\beta(v|s')dv$ is strictly single-crossing in $s'$. This implies that funded equilibria is completed characterized by the fixed points $s^* < 1$ of $\Phi$.

**Claim 1** $\Phi(\cdot)$ is continuous. $\Phi(s) = 0$ when $v^f(s) = 1$.

Given any $s \in [0, 1]$. Suppose $v(s) < 1$. Suppose $\Phi(s) \in (0, 1)$. Then for all $\epsilon > 0$,

$$\int_{v^f(s)}^1 (v - p)\beta(v|\Phi(s) + \epsilon)dv > 0$$

and

$$\int_{v^f(s)}^1 (v - p)\beta(v|\Phi(s) - \epsilon)dv < 0,$$

Then by the continuity of $v^f(\cdot)$ and the integral with respect to the lower limit of integration, there exists a neighborhood of $s$, $B_\delta(s)$ such that when $s' \in B_\delta(s)$,

$$\Phi(s') \in B_\epsilon(\Phi(s)).$$

The case $\Phi(s) \in \{0, 1\}$ is treated similarly.

Suppose $v^f(s) = 1$. Then by definition of $\Phi(\cdot)$, $\Phi(s) = 1$. If there exists $\epsilon > 0$ such that $v^f(s - \epsilon) = 1$, then $\Phi(s') = 1$ for all $s' > s - \epsilon$. If $v^f(s - \epsilon) < 1$ for all $\epsilon > 0$, since $p < 1$ and $v^f(\cdot)$ is continuous, choose $\delta$ such that for all $s' > s - \delta$, $v(s') > p$. Then $\Phi(s') = 1$ for all $s' > s - \delta$.  

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26There may be multiple unfunded equilibria, which are not captured by $\Phi$.

27It is essential that $p < 1$. If $p \geq 1$, $\Phi(\cdot)$ will be discontinuous: the backers will not contribute whenever $v^f(s) < 1$ ($\Phi(s) = 1$), and will contribute ($\Phi(s) = 0$, by definition of $\Phi$) whenever $v^f(s) = 1$. 

24
**Claim 2** \( \Phi(\cdot) \) is decreasing.

Let \( s_1 < s_2 \). When \( \Phi(s_1) = 1 \) then trivially \( \Phi(s_2) \leq \Phi(s_1) \). When \( \Phi(s_1) \in (0, 1) \), then by definition of \( \Phi \) we have

\[
\int_{v^f(s_1)}^{1} (v - p)\beta(v|\Phi(s_1))dv = 0
\]

and that \( v^f(s_1) < 1 \). This also implies \( v^f(s_1) < p \) (else \( \Phi(s_1) = 0 \) because the backer is never going to lose.).

Hence

\[
\int_{v^f(s_2)}^{1} (v - p)\beta(v|\Phi(s_1))dv \geq 0.
\]

So by definition \( \Phi(s_2) \leq \Phi(s_1) \).

If \( \Phi(s_1) = 0 \) then, since \( v^f(s_2) \geq v^f(s_1) \), \( \int_{v^f(s_2)}^{1} (v - p)\beta(v|0)) \geq 0 \). So \( \Phi(s_2) = 0 \) as well.

**Claim 3** \( \Phi(s) = 0 \) when \( s \) is sufficiently high. Since \( T \geq k - a > 0 \), whenever \( s \) is sufficiently high \( v^f(s) = 1 \).

By Claim 1,2,3, \( \Phi(\cdot) \) has a fixed point \( s^* \in [0, 1) \). The funded equilibrium is given by \( \sigma_{s^*} \), and the project is funded when \( v \in B^\sigma = [v^f(s^*), 1] \).

For any cutoff strategy \( \sigma \), \( U^{F\text{lex}}(s; \sigma) \) is single-crossing because of MLRP.\(^{28}\) Therefore, the best response to any cutoff strategies \( \sigma \) is also characterized by a cutoff \( s^* \in [0, 1) \) such that the backers contribute when \( s > s^* \) and postpone purchase whenever \( s < s^* \). However, existence of equilibrium is subject to a more restrictive condition than fixed funding.

Next, we prove Theorem 1.

First, we will express the profits in equation (1) and equation (2) as the difference between social surplus and consumer surplus when \( \sigma \) is a cutoff strategy. For each \((F, T, p)\)

\(^{28}\)For an arbitrary \( \sigma \), single-crossing could fail: the other backers can coordinate to contribute only when \( s \) is low. Then a backer who contributes when observing a high signal will end up paying \( p \) while getting nothing in return because the project fails to be funded.
and each corresponding equilibrium $\sigma$ with cutoff $(v^*, s^*)$,

$$
\Pi^{Fix}(T, p; \sigma) = \int_{v^*}^{1} (p(1 - G(s^*|v)) + vG(s^*|v) - k)f(v)dv \\
\Pi^{Flex}(T, p; \sigma) = \int_{0}^{1} p(1 - G(s^*|v))f(v)dv + \int_{v^*}^{1} (vG(s^*|v) - k)f(v)dv
$$

Furthermore, the social surplus is given by

$$
SS = \int_{v^*}^{1} (v - k)f(v)dv = \int_{v^*}^{1} (1 - G(s^*|v))v + G(s^*|v)v - k)dF(v)
$$

The (ex-ante) consumer surplus under fixed funding and flexible are

$$
CS^{Fix}(T, p) = \int_{0}^{1} \left( \int_{s^*}^{1} \left( \int_{v^*}^{1} (\bar{v} - p)\beta(\bar{v}|s)d\bar{v} \right) \right) g(s|v)ds \\
CS^{Flex}(T, p; \sigma) = \int_{0}^{1} \left( \int_{s^*}^{1} \left( \int_{v^*}^{1} \bar{v}\beta(\bar{v}|s)d\bar{v} - p \right) \right) g(s|v)ds
$$

**Lemma A.2.** For any funded equilibrium, $\sigma$, under $(F, T, p)$,

$$
\Pi^{Fix}(T, p) = SS^{Fix}(T, p) - CS^{Fix}(T, p) \quad (4) \\
\Pi^{Flex}(T, p; \sigma) = SS^{Flex}(T, p) - CS^{Flex}(T, p; \sigma) \quad (5)
$$

**Proof.** We give a proof of (4), the computation leading to (5) is along the same line. Let $\sigma$ be a funded equilibrium under $(Fix, T, p)$.

Note that

$$
SS^{Fix} - \Pi^{Fix}(T, p; \sigma) \\
= \int_{v^*}^{1} (1 - G(s^*|v))vf(v)dv - \int_{0}^{1} p(1 - G(s^*|v))f(v)dv
$$

The consumer surplus can be written as

$$
CS^{Fix} = \int_{0}^{1} \left( \int_{s^*}^{1} \left( \int_{v^*}^{1} (\bar{v} - p)\beta(\bar{v}|s)d\bar{v} \right) \right) g(s|v)ds f(v)dv
$$
It then suffices to show that (6) and (7) are equal. To this end, using the definition of $\beta(\cdot)$, we have
\[
\int_0^1 \left( \int_{s^*}^1 \left( \int_{v^*}^1 \tilde{v} \beta(v|s) dv \right) g(s|v) ds \right) f(v) dv
= \int_0^1 \int_{s^*}^1 \int_{v^*}^1 \frac{1}{\tilde{v}} g(s|\tilde{v}) f(\tilde{v}) g(s|v) f(v) d\tilde{v} ds dv
= \int_{s^*}^1 \int_{v^*}^1 \tilde{v} g(s|\tilde{v}) f(\tilde{v}) \left( \int_0^1 g(s|v) f(v) dv \right) d\tilde{v} ds
= \int_{v^*}^1 \tilde{v} [f(\tilde{v})(1 - G(s^*|\tilde{v}))] d\tilde{v}.
\]
This completes the proof. \qed

Next, we show that, by switching to fixed funding, an entrepreneur can maintain (or increase) the social surplus and decrease the consumer surplus.

**Lemma A.3.** Suppose a project can be funded by $(\text{Flex}, T, p)$ with equilibrium cutoff $(s^*, v^*)$. Suppose $v^* > 0$. Then there exists $(T', p')$ with $T' > T, p' > p$ and $p' > T'$, such that the project can be funded by $(\text{Fix}, T', p')$ with the same equilibrium cutoff $(s^*, v^*)$. Suppose instead $v^* = 0$, then for any $0 < \epsilon < k$ there exists $(T', p')$ with $T' > T, p' > p$ such that the project is funded by $(\text{Fix}, T', p')$ with cutoff $(s', \epsilon)$ where $s' > 0$.

**Proof.** Let $(v^*, s^*)$ be the equilibrium cutoff under $(\text{Flex}, T, p)$.

For each $\tilde{p} \geq p$, define the function $s : [p, 1] \to [0, 1]$ as
\[
s(\tilde{p}) = \min \left\{ s : \int_{v^*}^1 (v - \tilde{p}) \beta(v|s) dv \geq 0 \right\},
\]
which is the signal making a buyer indifferent between contributing or wait when they expect the project is built if $v \geq v^*$. Note that $s(\cdot)$ is continuous and increasing.

Suppose first that $v^* > 0$. This will imply $s^* > 0$.\footnote{If $s^* = 0$ then everyone contributes, implying $v^* = 0$.} Therefore,
\[
\int_{v^*}^1 (v - p) \beta(v|s^*) dv > \int_{v^*}^1 v \beta(v|s^*) dv - p = 0.
\]
Thus $s(p) < s^*$. On the other hand, $\lim_{p \to 1} s(p) = 1$, hence the intermediate value theorem guarantees the existence of $p' \in (p^*, 1)$ with $s(p') = s^*$. Now define $T' = p'(1 - G(s^*|v^*))$, which ensures the expectation is correct, that is, when the cutoff on signal is $s^*$ and price is $p'$, the project is funded if and only if $v \geq v^*$. Then, $(s^*, v^*)$ is the equilibrium cutoff for fixed funding given $(T', p')$. Finally note that since $p' > p$,

$$T' > p(1 - G(s^*|v^*)) = T.$$ 

So $(T', p') \in C_a$.

Suppose $v^* = 0$ and $s^* > 0$. Pick an $\epsilon \in (0, k)$, let $p(\epsilon)$ be such that

$$\int_0^1 (v - p(\epsilon))\beta(v|s^*)dv = 0$$

It is straightforward to see that $p(\epsilon) > p$. Define $T(\epsilon) = p(\epsilon)(1 - G(s^*|\epsilon))$. Then the equilibrium cutoff under $(Fix, T(\epsilon), p(\epsilon))$ is by construction $(s^*, \epsilon)$. Again note that $(T(\epsilon), p(\epsilon)) \in C_a$ and that $p(\epsilon) > T(\epsilon)$ (since $s^* > 0$).

Suppose $v^* = 0$ and $s^* = 0$. Pick $\epsilon \in (0, k)$ and for each $\delta > 0$ let $p(\epsilon, \delta)$ be such that

$$\int_0^1 (v - p(\epsilon))\beta(v|\delta)dv = 0$$

Let $T(\epsilon, \delta) = p(\epsilon, \delta)(1 - G(\delta|\epsilon))$. Then the equilibrium cutoff under $(Fix, T(\epsilon, \delta), p(\epsilon, \delta))$ is, by construction, $(s^*, v^*) = (\delta, \epsilon)$. \hfill \Box

**Proof of Theorem 1.** Let $(s^*, v^*)$ be the cutoff of a funded equilibrium $\sigma$ for $(Flex, T, p)$. By Lemma A.2, it suffices to show that $SS^{Fix}(T', p') = SS^{Flex}(T, p; \sigma)$ and $CS^{Fix}(T', p') < CS^{Flex}(T, p; \sigma)$.

Suppose $v^* > 0$. By Lemma A.3 there exists $(T', p') \in C_a$ that supports the same cutoffs as a funded equilibrium in fixed funding.

First observe that since the cutoffs are the same, the social surplus remain the same. Second,

$$CS^{Fix}(T', p') - CS^{Flex}(T, p; \sigma)$$

$$= \int_0^1 \int_{s^*}^1 \left( p - \int_{v^*}^1 p'\beta(\bar{v}|s)d\bar{v} \right) g(s|v)df(v)dv$$

28
Since by construction $U^{Fix}(s^*) = U^{Flex}(s^*)$, and $\beta(\tilde{v}|s)$ satisfies MLRP,

$$p - \int_{v^*}^{1} p' \beta(v|s) dv < 0$$
as $s > s^*$. So $CS^{Fix}(T', p') \leq CS^{Flex}(T, p; \sigma)$.

Suppose $v^* = 0$ and $s^* > 0$. Consider an $\epsilon$ such that

$$\int_{\epsilon}^{1} (v-k)f(v) dv > \int_{0}^{1} (v-k)f(v) dv.$$

Let $(T', p') = (T(\epsilon), p(\epsilon))$ be given by Lemma A.3. Then by construction $SS^{Fix}(T', p') > SS^{Flex}(T, p; \sigma)$ and by the same argument as above $CS^{Fix}(T', p') < CS^{Flex}(T, p; \sigma)$.

Suppose $v^* = 0$ and $s^* = 0$. The adjusted campaign in Lemma A.3. would then generate higher social surplus and lower consumer surplus, again leading to higher profit.

Proof of Theorem 2. Let $v^*(T, p), s^*(T, p)$ denote the cutoff of the funded equilibrium under $(T, p) \in C_a$ whenever the equilibrium exists. We establish the result for $a \in [0, k)$ by three claims.

Claim 1 Suppose $(T, p) \in C_a$ maximizes $\Pi^{Fix}$, then $v^*(T, p) \geq k$.

Suppose $v^*(T, p) < k$. The entrepreneur can pick some $p' > p$ such that $v^*(T, p') \in (v^*(T, p), k)$ and that $s^*(T, p') > s^*(T, p)$. Such $p'$ exists because $v^*(T, p)$ is continuous on $p \in (T, 1)$, by the construction of this fixed point in Proposition 3.1. Also, whenever the equilibrium $v^*$ becomes higher due to price increase, corresponding $s^*$ must rise (Otherwise $v^*$ will be lower due to increased price and increased number of contributors).

This process raises social surplus and decreases consumer surplus. To see this, let $U'(s) = \int_{v^*(T, p')}^{1} (v-p') \beta(v|s) dv$ and $U(s) = \int_{v^*(T, p)}^{1} (v-p) \beta(v|s) dv$. Let

$$h(v) = \begin{cases} 
  p^*, & v > v^*(T, p') \\
  v, & v^*(T, p) \leq v \leq v^*(T, p'). 
\end{cases}$$

We thus have $U'(s^*(T, p')) - U(s^*(T, p')) < 0.$ Moreover, for all $s > s^*(T, p')$, because

\[30\] The first term is zero, the second term is the expected utility for contribution evaluated at a signal higher than the equilibrium cutoff $s^*(T, p)$. 

29
\( h(\cdot) \) is increasing and \( \beta(\cdot|s) \) satisfies MLRP,

\[
U'(s) - U(s) = \int_{v^*(T,p')}^1 (v - p') \beta(v|s) dv - \int_{v^*(T,p)}^1 (v - p) \beta(v|s) dv
\]

\[
= - \int_{v(T,p)}^1 (h(v) - p) \beta(v|s) dv < 0.
\]

Finally note that consumer surplus is

\[
CS(T,p') = \int_0^1 \int_{s(T,p')}^1 U'(s) g(s|v) ds f(v) dv
\]

\[
CS(T,p) = \int_0^1 \int_{s(T,p)}^1 U(s) g(s|v) ds f(v) dv.
\]

Hence \( CS(T,p) - CS(T,p') > 0 \).

This shows Claim 1.

Claim 2 Suppose \((T,p) \in C_a\) maximizes \( \Pi^{Fix} \), then \( T = k - a \).

If \( s^* = 0 \) then \( v^* = 0 \) and \((T,p)\) is not optimal by Claim 1. So assume \( s^* > 0 \).

Suppose \( T > k - a \). Choose \( T' = k - a =< T \). For each \( \bar{p} \in [p,1) \), define

\[
s(\bar{p}) = \min \{ s : \bar{p}(1 - G(s|v^*(T,p))) = T' \}.
\]

Then \( s(\cdot) \) is continuous and \( s(p) > s^* \) because \( T' < T \) and \( s^* > 0 \). Since \( T' > 0 \), \( s(\cdot) < 1 \).

Define the continuous function \( J : [p,1) \to [0,1] \) as

\[
J(\bar{p}) = \int_{v^*}^1 (v - \bar{p}) \beta(v|s(\bar{p})) dv.
\]

Since \( s(p) > s^* \) and \( s^* \) is an equilibrium cutoff, \( J(p) > 0 \). Since \( s(1) < 1 \), \( J(1) < 0 \). Hence intermediate value theorem implies the existence of some \( p' \in (p,1) \) such that \( J(p') = 0 \).

By construction, funded equilibrium under \((T',p')\) exists, with cutoff \((v^*,s(p'))\), and \( s(p') > s^*(T,p) \) because \( p' > p \) and \( T' < T \). This implies that the social surplus under \((T',p')\) is the same as that under \((T,p)\) while the consumer surplus decreases due to a higher price. Hence \( T > k - a \) is non-optimal.

Claim 3 For \( T = k - a \), there exists \( \epsilon \) such that if \( T \leq p < T + \epsilon \), then \((k-a,p)\) is not optimal.
To see this, we show the following:

$$\limsup_{p \to T} v^*(T, p) < T.$$ 

Suppose there exists a sequence \( \{p_n\} \) with \( p_n > T \) and \( \lim_{n \to \infty} p_n = T \) such that \( \lim_{n \to \infty} v^*(T, p_n) \geq T \). Then there exists some \( N \) such that \( p_n < v^*(T, p_n) \) for \( n > N \). By Proposition 3.1, \( v^*(T, p_n) < 1 \). Hence for \( n > N \),

$$\int_{v^*(T, p_n)}^1 (v - p_n) \beta(v|s) dv \geq 0$$

for all \( s \), which implies \( v^*(T, p_n) > 0 \) is not an equilibrium cutoff (since everyone would like to contribute), a contradiction.

Suppose instead \( \lim_{n \to \infty} v^*(T, p_n) = T < 1 \), dominated convergence implies

$$\lim_{n \to \infty} \int_{v^*(T, p_n)}^1 (v - p_n) \beta(v|s(T, p_n)) dv = \int_{v^*(T, 0)}^1 (v - T) \beta(v|0) dv > 0$$

where the last inequality follows from the assumptions on \( g(s|v) \). This again implies existence of some \( n \) such that

$$\int_{v^*(T, p_n)}^1 (v - p_n) \beta(v|s) dv \geq 0$$

for all \( s \), so the cutoff \( v^* \) cannot be positive, a contradiction.

By the three claims,

$$\max_{(T, p) \in C_a} \Pi^{Fix}(T, p) = \max_{p \in [k - a + \epsilon, 1]} \Pi^{Fix}(k - a, p)$$

Since \( \Pi^{Fix} \) as a function of \( p \) is continuous over \([k - a + \epsilon, 1]\), maximum exists.

This finishes the proof when \( a \in [0, k) \).

Now we show how to approximately extract full surplus when the entrepreneur has asset \( k \) to fund the project.

Let \( \{s_n\} \) be a sequence converging to 1. Define \( p_n \) to be such that

$$\int_k^1 (v - p_n) \beta(v|s_n) dv = 0,$$

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so \( \{p_n\} \) is a bounded sequence. Define \( T_n = p_n(1 - G(s_n|k)) \).

By construction funded equilibrium under \((T_n, p_n)\) exists, with cutoffs \((v^*, s^*) = (k, s_n)\) for each \(n\). Moreover, seller’s profit is

\[
\int_k^1 (p_n(1 - G(s_n|v)) + G(s_n|v)v - k)f(v)dv.
\]

Since \(s_n \to 1\) and \( \{p_n\} \) is bounded, the profit converges to the maximal social surplus. \(\square\)

### B Equilibrium Existence for Flexible Funding

This section shows the condition under which a cutoff equilibrium in flexible funding exists.

**Proposition B.1.** Given \((\text{Flex}, T, p)\) with \((T, p) \in C_a\). A funded equilibrium in cutoff strategies exists if and only if

\[
\exists v \text{ s.t. } T = p(1 - G(s(v)|v)),
\]

where

\[
s(v) = \min \left\{ s : \int_v^1 \tilde{v} \beta(\tilde{v}|s)d\tilde{v} - p \geq 0 \right\}
\]

is the backers’ cutoff if they expect the project to be funded when value is above \(v\).

The condition of Proposition B.1 says that there exists \(v\) such that the expectation is correct and backers are responding optimally to the expectation. It also implies that only when

\[
k - a < \sup_{p \in [0,1]} \sup_{v \in [0,1]} p(1 - G(s(v)|v)) < 1^{31},
\]

would a funded equilibrium exist.

**Proof of Proposition B.1.** Assumptions on \(\{g(s|v)\}\) implies \(s(\cdot)\) is continuous when \(p < 1\). Moreover, when \(v = 1\), \(s(v) = 0\) by definition, so \(p(1 - G(s(1)|1)) = 0\). Since

\[31\text{The two terms } p \text{ and } 1 - G(s(v)|v) \text{ can not simultaneously be one, so it must be less than one.} \]
\( T \leq \max_v p(1 - G(s(v)|v)) \), by continuity of \( p(1 - G(s(v)|v)) \) with respect to \( v \) there exists \( v^* < 1 \) such that

\[
T = p(1 - G(s(v^*)|v^*)).
\]

Then by construction \( \sigma_{s(v^*)} \) is a funded equilibrium. Conversely, for any funded equilibrium with cutoff \( s^* \) and \( v^* \), it must be that

\[
T = p(1 - G(s(v^*)|v^*))
\]

and that \( s(v^*) = s^* < 1 \), which also implies \( p < 1 \). \qed