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Security Creation Costs and Economic Development*

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Abstract

We describe a tractable general equilibrium environment where producers can transform cash-flows at a cost to create securities that cater to the needs of heterogeneous investors. We use the resulting model to characterize the theoretical implications of reductions in the cost of cash-flow transformation activities. Those reductions result in a greater volume of cash-flow transformation but have ambiguous effects on capital formation, output and TFP, in clear contrast to the outcome of traditional financial development exercises.

Keywords: Endogenous Security Markets; Financial Engineering; Macroeconomic Aggregates

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1 Introduction

A vast literature – see e.g. Goldsmith (1969), McKinnon (1973), Shaw (1973), and King and Levine (1993) – has documented that financial and economic development are highly correlated across countries.\(^1\) This empirical regularity has spurred a structural literature that models and quantifies the connection between finance, investment, innovation and resource allocation.\(^2\) The typical exercise in that literature models explicit disruptions – limited enforcement and/or asymmetric information frictions – to the financial process that limits producers’ access to finance and then studies the consequences of gradually mitigating those frictions. As financing constraints become less severe, producers operate closer to their optimal scale and resources are more likely to be directed to their most productive use. This results in more investment, higher productivity, and more output. The size of those effects is a matter of debate,\(^3\) but there is little disagreement on the theoretical implications of financial development in this traditional sense.

As documented for instance by Greenwood and Scharfstein (2013), the size of the financial sector has increased dramatically in the United States and other industrialized nations over the past three decades. One of the main drivers of the growth of credit to the private sector has been the rise of asset-backed securitization activities whereby cash-flows generated by the

\(^1\)Rajan and Zingales (1998) use industry-level data to provide more evidence that causation runs, at least in part, from financial development to economic development.

\(^2\)In Aghion et. al (2005), for instance, agency problems limit innovators’ access to finance which dampens long term growth. Banerjee and Newman (1993), Galor and Zeira [1993], Aghion and Bolton [1997] and Piketty [1997] argue that similar agency problems can cause poverty traps. Quantitatively, Amaral and Quintin (2010) argue that this type of frictions alone could account for much of the development gap between the United States and middle-income nations such as Mexico or Argentina. Midrigan and Xu (2014) find that these frictions have a lower impact once agents are given more time to self-finance to mitigate the impact of the borrowing constraints they face, but Moll (2014) argues that the mitigating effects of self-financing depend critically on the nature of the idiosyncratic shocks producers face. For similar exercises, see e.g. Erosa (2001), Jeong and Townsend (2007), Erosa and Cabrillana (2008), Quintin (2008), Buera, Kaboski, and Shin (2011), Buera and Shin (2013), Caselli and Gennaioli (2013). Papers that study the finance-development nexus qualitatively include Greenwood and Jovanovic (1990), Bencivenga and Smith (1991), Khan (2001) and Amaral and Quintin (2006). Hopenhayn (2014) provides a detailed review of the literature on finance and misallocation.

\(^3\)See the contrasting views of Midrigan and Xu (2014) on one side and Moll (2014) and Amaral and Quintin (2006), on the other.
private sector are pooled and tranched into securities that cater to the needs of heterogeneous investors. This type of financial development is fundamentally different from the access-to-credit channel emphasized by the aforementioned literature, but it too could have an impact on the real economy. To see this, consider an economy that contains agents who, by taste or by constraint, only want to invest in safe securities. Without some financial transformation, the capital these agents are able to provide cannot be tapped to finance risky investment projects. By creating new securities that transform risky cash flows into securities with different characteristics, financial intermediaries allow heterogeneous agents to combine their resources and fund projects whose fundamental characteristics may not meet the particular needs of any specific type of investor. Cash-flow transformations makes it possible to activate projects that could not be funded otherwise.

All else equal, a fall in security creation costs should lead to more spending on the securities created by the production sector. Our goal of this paper is to characterize the theoretical implications of the resulting increase in financial investment for capital formation, output or TFP. We argue analytically and via numerical examples that a large share of the investment boom caused by cheaper security creation costs may be dissipated into security creation costs and producer/intermediation rents rather than spent on actual capital formation. As a result, the theoretical effect of increases in security creation activities on GDP is likely small and may well be negative, in direct contrast to the outcome of typical exercises in financial development.

As for aggregate TFP, costly security creation booms can cause productivity declines. The key reason for this is that, in our model, all active projects operate at their optimal scale. Our producers are not borrowing constrained: they can borrow as much as they want at equilibrium prices. A reduction in security creation costs does lead to the entry of producers whose projects were not profitable until it became cheaper to sell different parts of the associated cash-flows to different investors. But there is no reason to expect that those new producers are high TFP-producers. In an environment where all projects are operated at, or near, their optimal scale, high-TFP producers tend to be profitable whether or not the
cash-flows they generate can be repackaged. The producers that become active as a result of increased financial engineering tend to be low average productivity producers.

This aspect of our environment is in sharp contrast to what emanates from traditional models of misallocation, as described for instance by Hopenhayn (2014). In those models, mitigating financial disruptions allows producers to operate closer to their optimal scale, which drives wage rates up and causes low-productivity managers to exit. Both aspects – projects operating closer to their optimal scale and the exit of less productive managers – result in higher TFP, as it is conventionally measured. In our model, lowering security creation costs allows previously infra-marginal producers to become profitable, which tends to lower TFP. Securitization booms, that is, can be bad for aggregate productivity.

We formalize these ideas in an extension of Allen and Gale (1988)'s optimal security design model in which the production side of the economy aggregates up to a standard neoclassical production function. The economy contains agents who are risk-neutral and other agents who are highly risk-averse and have a high willingness to pay for safe securities. As in Allen and Gale (1988), existence boils down to a fixed point problem over state prices. Taking state prices as given, producers choose what menu of securities to issue. Given the resulting menu, investors choose a consumption policies which implies their willingness to pay for marginal increments in the supply of securities hence pin down state prices. The state prices assumed by producers must coincide with the state prices implied by portfolio optimization on the part of investors. Allen and Gale (1988) prove that the resulting fixed point problem always has a solution. We adapt their argument to establish the same result in an economy in which the market for physical capital and labor must clear.

In the economy we describe, absent security creation costs, it would be optimal for producers to sell the safe part of the stochastic cash-flows they generate to risk-averse agents and the residual claims to risk-neutral agents. But splitting cash-flows in this fashion is costly. As a result of these costs, some potentially profitable projects are left inactive, which results in less investment broadly defined. Our simple model makes it possible to characterize the theoretical implications of varying security creation costs. Our main finding is that while the
implications for overall investment are clear, the impact of changes in security creation costs on output and average productivity is fundamentally ambiguous.

2 The environment

Consider an economy with two dates, hence one period, populated by a large mass of producers, a mass $N > 0$ of workers, and unit masses of investors of two distinct types.\(^4\) The economy is subject to aggregate uncertainty which, for simplicity, is binary. With probability $\pi_g$, aggregate conditions are good while with probability $\pi_b$ they are bad.

Workers supply their unit endowment of labor for a competitively determined wage $w$ and consume their labor income at the end of the period. Producers, for their part, can each choose to operate a project whose activation requires an investment of one unit of the consumption good at the start of the period. An active project operated by a producer of skill $z > 0$ yields gross output

$$z^{1-\alpha}An^\alpha$$

at the end of the period, where $\alpha \in (0, 1)$, $n$ is the quantity of labor employed by the project, and $A \in \{A_b, A_g\}$ is a random shock common to all projects whose value depends on aggregate conditions.

Let $\mu$ denote the Borel distribution of producer types. To simplify our upcoming existence proof we assume that $\mu$ is defined by a continuous density function, although allowing $\mu$ features accumulation points would not change any of our results. We also assume that producer types are public information.

Producers must decide whether to activate their project before the aggregate shock is revealed, but choose how much labor to employ after $A$ is drawn. Given prevailing wage $w$
and aggregate shock $A \in \{A_b, A_g\}$, an active producer of type $z$ generates

$$\Pi(A, z, w) = \max_{n \geq 0} z^\alpha A n^\alpha - nw$$

in net operating income and we will write $n^*(A, w, z)$ for their optimal employment level. Our functional form for output implies that both net operating income and employment policies are linear in producer talent $z$.

Producers have no endowment at the start of the period hence must fund their project by selling promises to investors. Investors of type $i \in \{a, n\}$ – $a$ for risk-averse, $n$ for risk-neutral – are each endowed with a quantity $k_i$ of the good at the start of the period. Producers issue claims to their stochastic end-of-period output to investors. Selling one type of security is free, but selling two different types of securities carries a fixed cost $\zeta > 0$. This security creation cost for producers who choose to create distinct securities meant for each type of investors is the basis for the main comparative exercise we perform in this paper. One interpretation of this cost is that the two investor types are physically separated from one another. Producers must decide whether to locate near one type or near the other. Delivering payoffs to the closer type is free – this is a mere normalization– delivering payoffs to the more distant type is costly. A more general interpretation is that selling securities in two distinct markets – selecting a more complex capital structure, in other words – is more expensive that relying on just one source of external funds.

Producers are small hence, when considering which securities to issue, they take as given investors’ willingness to pay for marginal investments in the associated payoffs. Formally, let $q_n(x_b, x_g)$ be the price at which a marginal amount of a security with payoffs $(x_b, x_g) \geq (0, 0)$ can be sold to type-$n$ investors, where payoffs may depend on whether aggregate conditions are bad or good. Similarly, let $q_A$ be the price at which contingent securities can be sold to type-$a$ investors. Active producers choose non-negative security payoffs $(x_b, x_g) > (0, 0)$ and
consumption profiles \((c_1^p, c_2^p)\) to maximize

\[ c_1^p + \epsilon E(c_2^p) \geq (0, 0) \]

subject to:

\[
c_1^p = q_a(x_a^a, x_g^a) + q_n(x_b^n, x_g^n) - 1 - \zeta_1\{x_a^a \neq 0, x_n^a \neq 0\}
\]

\[
x_b^a + x_b^n + c_{2,b}^p \leq \Pi(A_b, w(b); z),
\]

\[
x_g^a + x_g^n + c_{2,g}^p \leq \Pi(A_g, w(g); z),
\]

where the indicator \(1_{\{x^a \neq 0, x^n \neq 0\}}\) takes value one when a non-zero payoff is sold to each investor type and \(\epsilon > 0\) is the rate at which producers discount end-of-period consumption. Here, \(c_1^p\) denotes the producer’s consumption at the start of the period while \(c_2^p\) is consumption at the end of the period. We will think of \(\epsilon\) as a small but positive number so that producers have a strong preference for early consumption. In Allen and Gale (1988) producers only value early consumption which is the limit case of our economy. A small but positive \(\epsilon\) will simplify the analysis of the the producer problem (see lemma 2.)

The restriction that security payoffs be non-negative is equivalent to banning short-sales. As Allen and Gale (1988) explain, limits on short-sales are necessary for equilibria to exist in this environment. In particular and critically for our purposes, no equilibrium with costly cash-flow transformation can exist without limits on short-sales since short-sales would enable producers to expand the supply of securities for free. Simply put, splitting cash flows cannot be profitable in a version of the economy we have described with unlimited short-sales since the resulting profits would be arbitraged away and the Modigliani-Miller theorem would have to hold. As Allen and Gale (1988) also explain, an outright ban on short-sales is not necessary for cash-flow transformation to be profitable, short-sales simply need to be costly enough.

Clearly, producers become active whenever non-negative consumption is feasible given the unit capital requirement and pricing kernels \(q_a\) and \(q_n\). We will write \(Z \subset \mathbb{R}_+\) for the Borel
subset of active producers.

Denote by \( x^i(z) \) the security payoff vectors created for investors of type \( i \in \{a, n\} \) by a producer of type \( z > 0 \). Investors take as given the set of securities available at the start of a particular period. From their point of view, the menu of securities is a set of gross returns

\[
R^i_j(z) = \frac{x^i(z)}{q_n(x^i_n, x^i_g)}
\]
on the securities issued by producers who choose to be active (i.e. whose \( z \in Z \)) for \( j \in \{b, g\} \).

We assume for simplicity that investors only consume at the end of the period although no result of substance would change if they valued consumption at the start of the period as well. Investors of both types thus invest their entire wealth in available securities at the start of the period, and their problem thus boils down to selecting what amount \( \eta(z) \) to spend on the securities created by each type of producer. Specifically, risk-neutral households solve:

\[
\max_{\eta \geq 0} \pi_b \int_Z \eta(z)R^a_b(z)d\mu + \pi_g \int_Z \eta(z)R^a_g(z)d\mu
\]
subject to

\[
\int \eta(z)d\mu = k_n.
\]

Likewise, risk-averse agents solve

\[
\max_{\eta \geq 0} \min \left\{ \int_Z \eta(z)R^a_b(z)d\mu, \int_Z \eta(z)R^a_g(z)d\mu \right\}
\]
subject to

\[
\int_Z \eta(z)d\mu = k_a.
\]

In other words, agents of type \( a \) are infinitely risk-averse in the sense that they only care about the worst-case scenario. While finite risk aversion would lead us to similar eventual
conclusions, Leontieff preferences have simplifying advantages for our purposes. To see why, let

$$R_a = \min \left\{ \frac{c_b^a, c_g^a}{k^a} \right\}$$

be the effective return these agents realize on their portfolio at the optimal solution to their problem, where

$$\left( c_b^a, c_g^a \right) = \left( \int_Z \eta(z)R_b^a(z)d\mu, \int_Z \eta(z)R_g^a(z)d\mu \right)$$

is their consumption profile. If $c_b^a < c_g^a$ at the optimal solution for risk-averse investors, their willingness to pay for a marginal investment in a security with payoffs $x_b, x_g$ must be

$$q_a(x_b, x_g) = \frac{x_b}{R_A}.$$  

Indeed, they only only value marginal payoffs in the lowest consumption state in that case. The symmetric property must hold when $c_b^a > c_g^a$. When $c_b^a = c_g^a$,

$$q_a(x_b, x_g) = \frac{\min(x_b, x_g)}{R_a}.$$  

For their part, letting

$$\bar{R}_n = \sup_{z \in Z} \pi_b R_b^n(z) + \pi_g R_g^n(z)$$

denote the highest expected return they can earn, risk-neutral investors are willing to pay

$$q_n(x_b, x_g) = \frac{\pi_b x_b + \pi_g x_g}{\bar{R}_n}$$

for any security with payoff $(x_b, x_g)$.

An equilibrium in this environment is a pair $(w_b, w_g)$ of wages of each possible realization of the aggregate shock, a subset $Z$ of active producers, their security policies $(x^n, x^a)$ and the associated returns $\{R^i(z) : i \in \{a, n\}, z \in Z\}$, investor consumption profiles $(c^a, c^n)$ and the associated pricing kernels $(q_a, q_n)$, producer consumption profiles $(c_1^a, c_2^a)$ and worker
consumption profiles \((c^w_b, c^w_g)\) such that, given prices:

1. Consumption policies solve each investor’s and each worker’s problem. In particular,
   \( (c^w_b, c^w_g) = (w_b, w_g) \);

2. Security policies solve each producer’s problem;

3. The labor market clears for each possible realization of the aggregate shock:
   \[
   \int_Z n^*(A_g, w_g, z) d\mu = \int_Z n^*(A_b, w_b, z) d\mu = N;
   \]

4. Security markets clear:
   \[
   k_a \bar{R}_a = \min \left( \int_Z \eta(z) R^a_b(z) d\mu, \int_Z \eta(z) R^a_g(z) d\mu \right),
   \]
   and \(k_n \bar{R}_n = \int_Z (\pi_b x_b^n(z) + \pi_g x_g^n(z)) d\mu;\)

5. The market for the consumption good clears at the beginning and at the end of the period:
   \[
   c^p_1 + \int_Z (1 + \zeta 1_{\{x^n \neq 0, x^n \neq 0\}}) d\mu = k_a + k_n,
   \]
   \[
   Nc^w_b + c^n_b + c^a_b = \int_Z z^{1-\alpha} A_b n^*(A_b, w_b, z) \alpha d\mu,
   \]
   and \(Nc^w_g + c^n_g + c^a_g = \int_Z z^{1-\alpha} A_g n^*(A_g, w_g, z) \alpha d\mu;\)
6. Security pricing kernels are consistent with investors consumption policies:

\[
q_a(x_b, x_g) = \begin{cases}
\frac{x_b}{\bar{R}_a} & \text{if } c^a_b < c^a_g \\
\frac{x_g}{\bar{R}_a} & \text{if } c^a_b > c^a_g \\
\frac{\min(x_b, x_g)}{\bar{R}_a} & \text{if } c^a_b = c^a_g
\end{cases}
\]

where \( \bar{R}_a = \min \left\{ \frac{c^a_b}{k^a}, \frac{c^a_g}{k^a} \right\} \)

\[
q_n(x_b, x_g) = \frac{\pi_b x_b + \pi_g x_g}{\bar{R}_n}
\]

where \( \bar{R}_n = \sup_{z \in Z} \pi_b R^n_b(z) + \pi_g R^n_g(z) \).

As we discussed in the introduction, an equilibrium in this class of environments with endogenously incomplete markets solves a fixed point problem on state prices. Producers take the pricing kernels \( q_a \) and \( q_n \) as given and select the menu of securities that maximize their profits. In turn, given available securities, investors choose an optimal consumption policies which implies pricing kernels. Equilibrium condition 6 states that the pricing kernels assumed by producers have to be consistent with optimal portfolio hence consumption choices by investors give the securities available to them. Allen and Gale (1988) show that an equilibrium exists in a version of this economy without explicit markets for capital and labor. Below we will adapt their argument to show that existence continues to hold in our neoclassical extension of their model.

3 Equilibrium properties

This section provides a proof that equilibria always exist in this environment and characterizes optimal financial policies. Allen and Gale (1988) establish existence in a model which nests ours on the investor preference side, but the production side of our economy features two factors of production whose markets must clear. In fact, our production side aggregates up to a standard neoclassical production function hence maps neatly into macroeconomic aggregates as they are conventionally measured.
3.1 Aggregation

Write
\[ K \equiv \int_Z d\mu = k_a + k_n - c_1 - \int_Z \zeta_{1\{x^n \neq 0, x^n \neq 0\}} d\mu \]
for the aggregate capital stock used in production in equilibrium. While the level of capital formation is an equilibrium object, the point of this section is that knowing that level is sufficient to calculate aggregate output. What is more, the relationship between capital formation and output assumes a traditional neoclassical form.

To see this, denote average producer productivity as
\[ E(z|z \in Z) \equiv \frac{\int_Z zd\mu}{\int_Z d\mu}. \]
It should be clear and we will soon establish formally (see lemma 3) that producer participation is characterized by a talent threshold \( \tilde{z} \) such that producers are active in equilibrium if and only if \( z \geq \tilde{z} \). Since each producer employs one unit of capital, we must have
\[ K = \int_{z \geq \tilde{z}} Z, \]
which pins down \( \tilde{z} \) given \( K \) hence, in turn, pins down average productivity. The fact than the marginal product of labor has to be equated across producers in equilibrium is then enough to compute output given \( K \) and aggregate conditions \( A \in \{A_b, A_g\} \), as the next result establishes.

**Lemma 1.** In any equilibrium, aggregate output hence aggregate consumption at the end of the period is given by:
\[ F(A, K, N) = E(z|z \in Z)^{1-\alpha} AK^{1-\alpha} N^\alpha. \]

**Proof.** Assume for concreteness that aggregate conditions are good so that \( A = A_g \). Algebra
in the other case is identical. Labor market clearing implies

\[ N = \int_{\mathbb{Z}} n^*(A_g, w_g, z) \]
\[ = n^*(A_g, w_g, 1) \int_{\mathbb{Z}} zd\mu \]
\[ = n^*(A_g, w_g, 1) KE(z|z \in Z). \] (3.1)

Indeed, the fact that producers all face the same wage hence choose the same marginal product of labor implies that labor choices are linear in producer skill. It follows that:

\[ F(A, K, N) = \int_{\mathbb{Z}} z^{1-\alpha} A n^*(A_g, w_g, z) \alpha d\mu \]
\[ = \int_{\mathbb{Z}} z A n^*(A_g, w_g, 1) \alpha d\mu \]
\[ = \int \mathbb{Z} z A \left( \frac{N}{KE(z|z \in Z)} \right) \alpha d\mu \]
\[ = \left( \frac{N}{KE(z|z \in Z)} \right)^\alpha \int \mathbb{Z} z d\mu \]
\[ = E(z|z \in Z)^{1-\alpha} A K^{1-\alpha} N^{1-\alpha}. \]

The second equality uses the fact that \( n^*(A_g, w_g, z) = zn^*(A_g, w_g, 1) \) for all \( z \in \mathbb{Z} \), while the third equality uses expression (3.1). This completes the proof.

One useful consequence of this aggregation result is that in any equilibrium

\[ w_b = \alpha E(z|z \in Z)^{1-\alpha} A_b \left( \frac{K}{N} \right)^{1-\alpha} < \alpha E(z|z \in Z)^{1-\alpha} A_g \left( \frac{K}{N} \right)^{1-\alpha} = w_g \]

so that knowing \( K \) is sufficient to infer not only \( E(z|z \in Z) \) but also both market clearing wages. In particular, the ratio of wages in the two aggregate states must be \( \frac{A_b}{A_g} \) in any equilibrium. Furthermore, our economy satisfies a traditional national accounting identity:

\[ N c_g^w + c_g^n + c_g^o = F(A, K, N). \]
Yet another consequence of this result is that Total Factor Productivity (TFP) as it is conventionally measured takes a simple form in this model, namely $E(z|z \in Z)^{1-\alpha}A$. TFP depends on the exogenous aggregate shock but also, endogenously, on the average skill of active producers. This will play a key role in some of comparative statics results.

### 3.2 Security markets

This subsection shows that when producers choose to bear the security creation cost $\zeta$, they do so for the purpose of issuing safe securities for which, in equilibrium, risk-averse investors are willing to pay a premium. This feature of our model accords well with the view associated for instance with Bernanke, Bertaut, DeMarco, and Kamin (2011) which attributes the recent increase in financial engineering activities to an increase in foreign appetite for safe US assets. Empirically, the primary goal and outcome of costly tranching activities seems to be to produce safe assets backed by risky collateral and this is precisely what happens in our model. We begin by establishing the fact that risk-averse investors, in equilibrium, only invest in risk-free securities, and pay a premium for those securities.

**Lemma 2.** In equilibrium, the consumption of risk-averse investors is risk free ($c^a_b = c^a_g$) and all securities created for those investors are risk-free as well:

$$x^a_b(z) = x^a_g(z) \text{ for all } z \in Z.$$

Furthermore, $\bar{R}_a < \bar{R}_n$ so that risk-neutral investors earn a higher expected return than risk-averse investors.

**Proof.** Assume, by way of contradiction, that $c^a_b > c^a_g$. Then risk-averse agents pay nothing for payoffs contingent on the bad state. But for $c^a_b > c^a_g$ to hold in equilibrium, a positive mass of producers must create securities with $x^a_b(z) > x^a_g(z)$. These producers would be better off consuming $x^a_b(z)$ (since $\epsilon > 0$) or selling that part of the payoff to risk-neutral agents, which is the contradiction we sought. If $\bar{R}_n \leq \bar{R}_a$ then it cannot be profitable for any producer to
issue securities to risk-averse agents. They would always be better off selling risk-free assets to risk-neutral investors and consuming any excess payoffs, for one option. This is incompatible with market clearing. This contradiction completes the proof.

A key part of each producer’s problem, then, is to decide what risk-free payoff \( x^a \) to promise risk-averse agents. That payoff must satisfy

\[
x^a \leq \Pi(A_b, w(b); z)
\]

for any producer \( z \). Indeed, risk-free promises have to be risk-free which means that producers must be able to deliver on them even in the worst-case scenario. When \( \epsilon \) is high enough, the optimal policy for producers may involve selling just enough securities to pay for the capital they need and consuming as much as they can at the end of the period. When, on the other hand, \( \epsilon \) is low enough – zero, in the limit – producers seek to maximize their consumption at the start of the period, as in Allen and Gale (1988). To economize on the cases we would need to consider, we will henceforth focus on on the case where \( \epsilon \) is vanishingly small.

In principle then, our economy comprises producers who issues safe securities only, producers who issue risky-securities only, and producers who bear the creation cost in order to issue both types of securities. Since profits increase linearly in \( z \), producer participation is governed by a talent threshold \( z \) above which it is profitable to install the required unit of capital. Whether or not producers choose to bear the security creation cost and issue both types of securities is governed by a second threshold, as we now establish.

**Lemma 3.** Producers activate their project if and only if \( z \geq z \) for a talent threshold \( z \geq 0 \). For \( \epsilon \) small enough, a second threshold \( \bar{z} \geq z \) exists such that producers bear the security creation cost and create two securities if and only if \( z \geq \bar{z} \). Furthermore, producers who issue safe securities always issue the most of it they can i.e.

\[
x^a(z) \in \{0, \Pi(A_b, w_b; z)\} \text{ for all } z \in Z.
\]
Proof. Since producers profits are linear in talent $z$ the existence of a participation threshold $z^*$ is obvious. For the second threshold, assume that $\epsilon = 0$. Active producers issue two securities rather than one when

$$\frac{\Pi(A_b, w_b; z)}{\bar{R}_a} + \frac{\pi_g \Pi(A_g, w_g; z) - \Pi(A_b, w_b; z)}{\bar{R}_n} - \zeta \geq \max \left\{ \frac{\Pi(A_b, w(b); z)}{\bar{R}_a}, \frac{\pi_g \Pi(A_b, w_b; z) + \pi_g \Pi(A_g, w_g; z)}{\bar{R}_n} \right\}. $$

Indeed, instead of selling two securities they could sell risk-free securities only, or sell everything to risk-neutral agents. Note that the two payoffs that involve the sale of risk-free security increase with the volume of these securities issued, so that producers always issue as much of them as they can ($x^a = \Pi(A_b, w_b; z)$) when they issue any amount at all.

Subtracting $\frac{\Pi(A_b, w(b); z)}{\bar{R}_a}$ from both sides of the inequality and rearranging gives two conditions:

$$\frac{(\Pi(A_g, w_g; z) - \Pi(A_b, w_b; z))}{\pi_g \frac{\Pi(A_b, w(b); z)}{\bar{R}_a}} \geq \zeta, \quad (3.2)$$

$$\frac{\Pi(A_b, w_b; z)}{\bar{R}_a} \left( \frac{1}{\bar{R}_n} - \frac{1}{\bar{R}_a} \right) \geq \zeta. \quad (3.3)$$

The first condition says that producers must be better off paying $\zeta$ to sell excess payoffs to risk-neutral agents while the second one says that extracting the safe part of producers’ output and selling it to risk-averse agents must cover the security creation cost. Since both left-hand sides rise linearly with $z$, it follows that if a producer of type $z$ finds it profitable to issue two securities, this is true for all producers of higher talent as well. This completes the proof.

This leaves one aspect of security policies to characterize. Do small producers – active producers whose talent is below the upper threshold $\bar{z}$ – choose to issue safe securities meant for risk-averse agents, or do they issue risky securities? We will now show by example that
either case is possible in equilibrium. To that end, we need to parametrize our model. We set \( A_b = 5 < 7.5 = A_g \) and make the probability that aggregate conditions are good equal to 90%. We set the labor share \( \alpha \) to 60%. We assume equal masses of the two types of investors while endowments are \( k_a = 0.4517 \) and \( k_n = 0.1683 \). Finally, we assume that the distribution of producer talent is log-normal with location parameter 0.1 and dispersion parameter 0.5, truncated to \([0, 1]\). We emphasize that this parameterization is simply meant to illustrate what financial policies can look like in equilibrium, we do not think of it as a careful calibration of our environment. Still, our simulations suggest that the qualitative properties we describe in this section are robust to even drastic changes in parameters.

In this parameterized version of our economy, the effect of varying \( \xi \) from zero to a level so high that no costly security creation takes place is shown on figure 1. As we will discuss in the next section, it is not possible to establish in full generality that equilibria have to be unique in our economy, but in all the cases shown on the figure our numerical search did only produce one equilibrium.

As \( \xi \) rises – as we move from left to right on the horizontal axis on panel A of figure 1 - security returns fall overall because it becomes more costly to produce them. Interestingly, the return on risky security is non-monotonic around \( \xi = 0.15 \). As long as \( \xi \) is below 0.15, all producers bear the tranching cost. At around \( \xi = 0.15 \) however, some producers choose to produce risky securities only as shown in panel D which causes an increase in the supply of risky securities and a decrease in the supply of safe securities. The spread between the two securities thus starts to increase noticeably.

In all our simulations we found that - given \( \xi \) and the spread between the two yields – either all producers who issue one security issue risky securities, or, instead, they all prefer to issue safe securities, or they are all exactly indifferent between the two types of securities. In particular, there does not exist a third threshold between \( \bar{\xi} \) and \( \tilde{\xi} \) such that producers in the two resulting intervals have different optimal security policies. As shown in panel B in the figure, it is nevertheless the case that past \( \xi = 0.3 \) some producers who issue only one type of security issue safe securities while other issues risky security. This because the
Figure 1: The effects of security creation costs
only equilibria we found in that region feature yields such that all producers in the \([z, \bar{z}]\) are exactly indifferent between the two types of securities. Security markets cannot clear unless different producers are allocated to different security policies in such a way as to exactly clear market.\(^5\)

Panels C of the figure confirms that as \(\zeta\) rises it becomes necessary for producers who do not bear the security creation cost to add to the supply of safe assets. As a result of this, the effect of greater security creation costs on yields is particularly pronounced for the safe yield. In particular, the spread between the two yields tends to rise as the security creation cost rises. This prediction arises in all the parameterizations we have considered, and we find it intuitive. Costly security creation is a technology that enables the extraction of safe securities backed by risky assets. The risk-premium that arises in our model is the compensation producers earn for bearing additional cost \(\zeta\). As \(\zeta\) rises, producers must be compensated at a higher rate. Not surprisingly then and as shown in panel D of the figure, the production of safe assets eventually falls drastically as \(\zeta\) rises.

### 3.3 Existence

Given the results we established in the previous section, an equilibrium boils down to rates of returns \((\bar{R}_n, \bar{R}_a)\) and wages \((w_b, w_g)\) such that security and labor markets clear. Classical arguments imply that such a combination of prices can always be found, as we now establish.

**Proposition 4.** An equilibrium exists.

**Proof.** Given household endowments \((k_n, k_a) > (0, 0)\), we need to find two security prices \((R_n, R_a)\) and two labor prices \((w_b, w_g)\) such that all markets simultaneously clear. Any guess for \((R_n, R_a, w_b, w_g) \in \mathbb{R}_+^4\) implies a corresponding set \(Z(R_n, R_a, w_b, w_g) \subset \mathbb{R}_+\) of active producers. We know from the previous section that this set is fully described by a simple threshold \(\tilde{z}\) and it should be clear that the threshold varies continuously with prices. Next, we

\(^5\)That randomization is necessary for existence in models with discrete choice is well-known. See for instance Amaral and Quintin (2010) or Halket (2014) for a discussion.
need to compute excess demand for each security type and labor for each of the two possible realizations of the aggregate shock in the current period. Starting with risky securities created for risk-neutral agents:

\[
ED_n(R_n, R_a, w_b, w_g) = k_n - \int_{Z(R_n, R_a, w_b, w_g)} \frac{E(x^n(z))}{R_n} d\mu,
\]

where \(E(x^n(z))\) is the expected payoff of risky securities created by producers of type \(z \in Z(R_n, R_a, w_b, w_g)\). As for risk-averse agents:

\[
ED_a(R_n, R_a, w_b, w_g) = k_a - \int_{Z(R_n, R_a, w_b, w_g)} \frac{x^a(z)}{R_a} d\mu,
\]

where

\[
x^a(z) \in \{0, \Pi(A_b, w_b; z)\}
\]

is the risk-free payoff selected by producers of type \(z \in Z(R_n, R_a, w_b, w_g)\). Excess demand for labor when the aggregate state is good is

\[
ED_L^g = \int_{Z(R_n, R_a, w_b, w_g)} n^*(A_g, w_g, z_g) d\mu - 1
\]

and the corresponding expression defines \(ED_L^b\) for the case where the aggregate shock is bad.

We need to prove that \((ED_n, ED_a, ED_L^b, ED_L^g)\) is zero for at least one four-tuple of prices.

Holding other prices fixed, each element of the \(ED\) demand vector is continuous (since \(\mu\) is defined by a continuous density function) and strictly monotonic in its own price. It also diverges without bound as each price goes to zero. Existence of the zero we need follows from classical arguments. To see this, for all \(n \in \mathbb{N}\), define \(A_n = \left[\frac{1}{n}, n\right]^4\). Then define \(G_n : ED(A_n) \mapsto A_n\) by

\[
G_n(y_1, y_2, y_3, y_4) = \arg \max_{R_n, R_a, w_b, w_g \in A_n} \frac{w_b y_3 + w_4 y_4 - R_n y_2 - R_a y_1}{R_n R_a}
\]
Roughly speaking, $G$ raises wages when there is an excess demand for labor and lowers rates of returns when there is an excess demand for securities. The Theorem of the Maximum implies that $G_n$ is non-empty, upper hemi-continuous and convex-valued. It follows that $G_n \times ED$ has a fixed point on $ED(A_n) \times A_n$.

Letting $n$ go to $+\infty$ gives a sequence of prices. That sequence must have a bounded subsequence. To see why, assume for instance that $R_a$ diverges to $+\infty$. Then at least one wage must fall to zero. Say that $w_B$ goes to zero. So, then, must $w_g$ since otherwise there would eventually be an excess supply of labor in the good state, which, given our mapping, would mean that $w_g$ follows $\frac{1}{n}$ at least along a subsequence. According to the same mapping, collapsing wages require that excess supply for labor remain positive in both states, which means that aggregate labor demand is bounded above, which means that profits are bounded above (since they are linear in the wage bill.). But then a diverging $R_a$ would imply that excess demand for safe securities is eventually positive, which would imply that $R_a$ eventually follows $\frac{1}{n}$ at least along a subsequence. Arguments are similar for the other three prices.

Since the sequence of fixed points above is bounded above and below, it must have a convergent subsequence. None of the associated prices can converge to zero. Assume for instance that $w_b$ did converge to zero. Given the mapping we have defined, this requires that aggregate labor demand remains below 1 in the bad state so operating profits in the bad state converge to zero. Since $w_g$ cannot diverge to $+\infty$ as established in the previous paragraph, excess labor demand in the good state has to be non-negative infinitely often which requires that $w_g$ also converges to zero at least along a subsequence. But then either return would have to diverge to infinity since otherwise with vanishing wages there would have to be an excess demand for labor eventually, which is incompatible with declining wages given our mapping. Again, the other prices can be dealt with using similar arguments.

It follows that along the convergent sequence of fixed point introduced above, the price

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6Otherwise, profits are bounded above unless demand for labor diverges to infinity (profits are linear in the wage bill) along a subsequence. If labor demand diverges, wages must converge to zero in at least one state.
part of the fixed point is eventually in the interior of $A_n$. But given the definition of $G_n$ this is only possible if all excess demands are zero. This completes the proof of existence. \[ \Box \]

As in Allen and Gale (1988) it is not possible to establish uniqueness at this level of generality. Yet, it is possible to obtain some sharp comparative statics implications, as we will now explain.

## 4 Comparative statics

This section characterizes the comparative statics properties of our model of cash-flow transformation. We are particularly interested in contrasting these properties with the theoretical predictions that emanate from traditional financial development exercises along the lines of Amaral and Quintin (2010) or Midrigan and Xu (2014).

In standard financial development exercises, producers face borrowing constraints caused by various market imperfections. These constraints limit participation by producers hence investment but they also prevent producers from operating on an optimal scale. Qualitatively, relaxing those constraints implies more lending, more capital formation and a higher TFP as capital becomes more concentrated in productive hands. While the size of effects is a matter of debate,\(^7\) there is no disagreement that qualitatively financial development moves output, capital use and TFP in the same direction.

The comparable exercise in our model is to reduce the security creation cost $\zeta$. Transforming cash-flows serves a fundamental purpose by creating securities that cater to the needs of heterogenous investors, but it is limited by security creation costs. The aggregation results we establish in section 3.1 immediately imply that output, capital formation and TFP cannot co-move following reductions in $\zeta$. Indeed, output and capital formation are pinned down by the lower participation threshold. They go up if and only if that threshold goes down. But

\(^7\)See the contrasting views of Midrigan and Xu (2014) on one side and Moll (2014) and Amaral and Quintin (2006), on the other.
TFP is likewise pinned downed by that threshold, and it must move in the opposite direction from output and capital formation.

The fundamental reason for this is that our producers are not borrowing constrained. When they choose to operate, they operate at their optimal scale. The marginal product of labor is equated across all production units. When there are output gains following a reduction in security creation costs, they are entirely due to capital deepening, and capital deepening requires the activation of marginal, low-productivity producers. To put this differently, producers who only become profitable following a reduction in security creation costs are marginally talented producers. When they start participating, they bring average productivity down. As we will discuss below, relaxing the assumption that capital use is the same across producers leads to more nuanced predictions for TFP.

The key question is should we expect capital formation to go up following a reduction in security creation cost? Take the case in which, at the initial equilibrium, $\bar{z} < \bar{z}$ so that some producers engage in costly security creation while others, including marginal producers, do not. Holding prices the same (abstracting from general equilibrium effects, that is), a reduction in $\zeta$ can only move the upper threshold. In particular, in partial equilibrium, reductions in $\zeta$ have no impact on output, capital formation and TFP. This stands in contrast to exercises in which borrowing constraints on producers are relaxed which has a direct impact on their output, even in partial equilibrium.

The fact that more producers engage in costly security creation does put pressure on security yields so that security and factor prices must change in equilibrium which in turn may move the participation threshold. Because this relies on price effects rather than direct effects, it seems natural to expect this particular effect to be quantitatively small. More strikingly, the direction of this effect is not clear a priori, as we will illustrate below.

In any equilibrium, we have

$$K = k_a + k_n - c_1^p - \int_{z \geq \bar{z}} \zeta d\mu.$$
Indeed, spending on securities must equal the sum $k_a + k_n$ of endowments. Good markets clearing implies that this total spending on securities must be split between capital formation ($K$), producer profits/rents $c_p$ and the aggregate cost $\int_{z \geq \bar{z}} \zeta d\mu$ of cash-flow tranching activities.

Our model’s structure is such that producer rents must increase as $\zeta$ falls. Indeed, the size of producer rents is a function of the dispersion in talent among active producers. If all producers were of the same talent, free entry would imply that all producers, not just marginal producers, earn zero profits. When producers are heterogeneous, profits are linear in producer talent.

When capital formation does go up (i.e. when the participation threshold falls), this must be because factor prices have fallen since marginal producers do not engage in costly security creation. So all previously active producers see their rents increase, an increase compounded by the reduction in $\zeta$ for the largest producers, so that total rents and the dispersion of those rents must increase when capital formation increases, and vice-versa.

The impact of changes in $\zeta$ on the resources $\int_{z \geq \bar{z}} \zeta d\mu$ spent on costly security creation, for its part, has to be non-monotonic, for a simple reason. It must be zero both when $\zeta = 0$ and when $\zeta$ is so large that no producer engages in costly security creation. There must be regions where $\int_{z \geq \bar{z}} \zeta d\mu$ is non-monotonic, therefore.

We will now show by way of example that this non-monotonicity can result in a non-monotonic relationship between security creation costs and output. To build such an example, we employ once again the parameterization discussed in section ?? . We continue to emphasize that this parameterization is only meant to produce a specific display of the theoretical relationships we have discussed in this section, not to serve as a careful quantitative evaluation of these relationships. But we did find the qualitative nature of the relationships we obtain in this example to be remarkably robust to even large changes in all these parameters.

Each panel on figure 2 shows the behavior of an equilibrium variable of interest as security creation costs go from zero to a value so high that the volume of costly security creation becomes negligible. As we mentioned in the previous section, we cannot establish theoretically
Figure 2: The effects of security creation costs
that equilibria are unique but in all the cases reported on the figure, our numerical procedure only found one equilibrium, even if started from very different initial conditions. Panel A shows that, no surprisingly, as $\zeta$ falls (as we move from right to left along the horizontal axis), the volume of securities created by producers who do bear the creation cost increases. That variable simply adds the two lines displayed in panel D of figure ???. Lowering security creation costs implies more costly security creation.

Panel B displays the non-monotonicity of the resources $\int_{z \geq \bar{z}} \zeta d\mu$ devoted to security creation activities. When $\zeta = 0$ no resources are used while when $\zeta$ becomes large ever fewer producers choose to bear the cost which brings total spending back down towards zero. Panel D of figure ?? shows that aggregate resources devoted to security creation costs begin to fall as $\eta$ rises precisely when some producers start producing safe securities only. Past that level of $\zeta$, costly cash-flow transformation is no longer a profitable way to increase the supply of safe assets, despite the continued fall in safe yields.

As panel C and D of figure 2 show, the non-monotonicity of security creation expenditures causes both capital formation and producer rents to be non-monotonic. As we explained above why producer rents and capital formation must co-move. Panels E and F show that given the behavior of capital formation, GDP and TFP must likewise be non-monotonic and move in opposite directions from one another.

5 Discussion of key assumptions

As we mentioned, we have found these qualitative properties to be robust to even big changes in parameters. Equally robust is the finding shown in panel E of figure 1 that when $\zeta$ is low, the relationship between security creation costs and output is negative. The calculations also turn out to be robust to taking into account the dynamic implications of lower security creation costs. We verified this by considering an overlapping generation version of our economy where investors work when young and invest their savings when old. Increases in output and wages increase investable wealth in the next period. In simulations of the stochastic
steady state of the resulting economy, we find in all our simulations that the static source of non-monotonicity we described in the previous section continue to dominate the behavior of macroeconomic aggregates, in part because the economy features very quick transitions for reasonable values of discount rates.

One aspect of our model which does plays a potentially big role in generating non-monotonicities in output is the assumption that capital use is homogeneous across producers. To make this point precise, consider a version of our model in which producers can select a scale \( k \geq k^0 \) on which to operate. Their gross output is

\[
z^{1-\alpha} A k^\eta n^\alpha,
\]

when aggregate conditions are \( A \) and they employ labor \( n \), where \( \eta > 0 \) and \( \alpha + \eta < 1 \). In this variation of our model, changes in \( \eta \) would cause a reallocation of capital across producers. Specifically, a decrease in \( \zeta \) means that more producers engage in costly security creation, as before. These producers see their average cost of capital go down hence increase their scale. What is more, the resulting pressure on factor prices causes marginal producers to exit, which helps average productivity.

In simulations of this version of our model available upon request, we find that non-monotonicities in output are less frequent and less pronounced when they do occur, not surprisingly. However, this depends critically on the assumption that producers can increase their capital scale without affecting the cost \( \zeta \) of security creation. When, instead, the cost of issuing two types of securities rises in proportion to the capital scale, we find non-monotonicities in capital formation and output appear to be as frequent as in the economy we described in the previous section.

Under either way of modeling security creation costs, the consequences of changes in security creation costs remain muted. The reallocation of capital that follows changes in \( \zeta \) only involve producers in the neighborhood of the upper talent threshold around which producers are indifferent between creating one and two securities. This local effect does not
translate to large aggregate effects.

6 Conclusion

We have described a general equilibrium model in which the production side of the economy aggregates up to a standard neoclassical production function but in which, on the finance side, producers have the option to transform cash-flows in order to create securities that cater to the needs of investors with heterogenous preferences. In our model as in recent data, the main purpose of costly cash-flow transformation is to create safe securities backed by risky assets, and to sell those securities at a premium.

Reductions in the cost of security creation result in a greater volume of cash-flow transformation. This bout of financial activity has ambiguous theoretical effects on capital formation, TFP and GDP. This is because the relationship between the resources devoted to security creation and security creation costs has to be monotonic. Total security creation costs are zero when cash-flow transformation is free and must return to zero once again when cash flow becomes prohibitively expensive.

While this theoretical possibility is interesting in its own right, we view the primary message of our experiments to be that bouts of securitization are very different from traditional periods of financial development caused by general improvements in the way financial markets function. As a result, it may be appropriate to think of financial development as consisting of two phases.\footnote{This two-stage view of financial development has a counterpart in the model of Acemoglu, Aghion, and Zilibotti (2006) in which, in a nutshell, economic development consists first of harvesting low-hanging fruits but becomes tougher to sustain once obvious growth opportunities have been implemented.} In emerging economies, institutional gains and the resulting gains in financial activity enable productive but previously constrained producers to become active and/or to operate more effectively. This results in potentially large economic development gains, as has been emphasized by the traditional literature on financial development. While the size of those gains is a matter of debate, there can be little disagreement on the direction of the effect during this initial phase of financial development.
Once higher levels of financial development are reached and markets function well, financial innovation tends to take the form of repackaging of fundamental cash-flows to create securities that appeal to the tastes of heterogenous investors. Not only are the output and productivity effects of this second phase likely to be small, they may well turn out to be negative.

References


