Financial Engineering and the Macroeconomy

Pedro Amaral (California State University, Fullerton)
Erwan Quintin (University of Wisconsin – Madison)
Dean Corbae (University of Wisconsin – Madison)

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Pedro S. Amaral  
California State University Fullerton  

Dean Corbae  
University of Wisconsin – Madison  

Erwan Quintin†  
University of Wisconsin – Madison  

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Abstract  

The volume of financial engineering has grown markedly over the past few decades as a result of technological improvements, regulatory arbitrage, and increased appetite for safe assets, inter alia. We describe a dynamic model of security creation where the impact of such changes can be characterized and quantified. These shocks can cause large increases in costly security creation volumes. But the resulting impact on output, capital formation and TFP is generally small, and may well be negative. Even combining an increase in appetite for safe assets with large external inflows of capital has a muted impact on macroeconomic aggregates. Financial engineering booms do have significant welfare consequences however, by changing the distribution of financial income and consumption across investor types. At the same time, the rents earned by the agents who engage in cash-flow transformation increase markedly, which provides a potential explanation for the vast increase in financial sector rents over the past few decades documented by Philippon and Reshef (2012).

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†Corresponding author. Erwan Quintin, 5257 Grainger Hall, 975 University Avenue, Madison, WI 53706. Phone/fax: 608 262 5126. Email: equintin@bus.wisc.edu.
1 Introduction

The volume of financial engineering – by which we mean the transformation of cash-flows to create securities that cater to the needs of heterogeneous investors – has grown markedly across the world over the past few decades. In the United States for instance, cash-flows created by the corporate sector such as receivables and business loans are now routinely pooled and tranched into securities with different risk and liquidity characteristics. Figure 1 illustrates the growing importance of financial engineering by plotting the outstanding volume of Asset-Backed Securities (ABS), excluding housing-related securities. It also shows that within this asset class, Collateralized Loan Obligations (CLOs) – which are securities backed by business loans – have grown from virtually non-existent in the mid-1990’s to over half a trillion dollars in 2017.\footnote{Although only shorter data are available for the Asset-Backed Commercial Paper (ABCP) market, we know that its volume doubled to reach over 1.2 trillion dollars between 2000 and 2007. That market collapsed in 2008 and has yet to recover. Figure 1 shows that the CLO market was much less affected by the crisis and has doubled in size since then.}

At least two concurrent phenomena have fueled the rise of financial engineering activities. First, technological improvements and regulatory arbitrage have made the activity cheaper.\footnote{See Allen and Gale (1994), for an early review of factors behind the boom in financial innovation over the past few decades.} Second, demand for the securities created via financial engineering – appetite for highly rated assets, in particular – has increased.\footnote{See Bernanke, Bertaut, Demarco, and Kamin (2011). As they put it, “Given the strength of demand for safe U.S. assets, it would have been surprising had there not been a corresponding increase in their supply.”} In this paper, we lay out an environment in which supply and demand shocks cause changes in the volume of costly security creation and use the resulting model to take on a simple question: What should we expect the impact of financial engineering booms to be on macroeconomic aggregates such as GDP, capital formation, and total factor productivity (TFP)?

Our model is a dynamic extension of Allen and Gale (1988)’s optimal security design model in which the production side of the economy aggregates up to a standard neoclassical model with aggregate uncertainty. The economy contains investors (households) who are
risk-neutral, as well as investors who are highly risk-averse and have a high willingness to pay for safe securities. Absent transaction costs, it would be optimal for producers to sell the safe part of the stochastic cash-flows they generate to risk-averse agents and the residual claims to risk-neutral agents. But splitting cash-flows in this fashion is costly. Given this cost, producers choose which securities to create taking their market value – i.e. the willingness by households to pay for these securities – as given. Given the resulting security menu at each possible history, households choose a consumption policy which, in turn, pins down their willingness to pay for securities. In equilibrium, the resulting pricing kernel has to coincide with the kernel assumed by producers. Allen and Gale (1988) show that this fixed point problem always has a solution in their static environment. We show that the same result holds in our dynamic framework.

We go on to fully characterize optimal security creation policies. First, it only makes sense to sell risk-free securities to risk-averse households, and producers who do so always issue as much of it as they can. Producers who issue safe securities either retain (consume, literally speaking, in our model) residual cash-flows or, instead, sell them to risk-neutral households when the value of doing so exceeds the security creation cost. In our model, as in recent US data, security creation activities result in the production of safe securities backed by risky assets. Not surprisingly then, we find that lowering security creation costs or increasing the fraction of risk-averse agents results in an increase in costly security creation activities and, in particular, increased issuance of safe securities.

While these supply and demand shocks can easily cause large increases in the volume of costly security creation, the resulting effect on macroeconomic aggregates turns out to be small. Keeping prices fixed, changes in the cost of security creation or in the fraction of risk-averse investors generally leave the set of active producers unchanged. This is because marginal producers – producers roughly indifferent between operating or not – are not large enough to justify bearing the security creation costs. It follows that only general equilibrium effects – endogenous changes in security returns and wages – affect the set of active producers following changes to the environment. This suggests that the connection between financial
engineering booms, output, capital formation and TFP is bound to be quantitatively small, a conjecture we verify via calibrated numerical simulations of our model.

Perhaps more surprisingly, the impact of supply or demand-driven security creation booms on capital formation and output can even be negative. In our model, spending on securities is divided between capital formation, producer rents, and security creation costs. While total spending on securities always rises following a decrease in the security creation cost or an increase in demand for safe assets, so can the resources spent on security creation as more producers engage in it. Capital formation, hence GDP, can fall. We also find that when average output and capital formation do rise as financial engineering activities increase, TFP tends to fall. Put another way, when more financial engineering is associated with more output, it tends to be associated with lower TFP, due to selection effects. This is because the increase in capital formation increases the participation of marginal producers, and those producers drag average TFP down.4

Demand-driven security creation booms are particularly likely to have a negative impact on output. When booms are caused by a decline in security creation costs, the smaller cost per producer offsets the fact that more producers choose to bear the security creation cost. When the shock comes from the demand side, that offsetting effect is no longer active and financial engineering booms must imply that more resources are spent on security creation.

Demand-driven and supply-driven booms in security creation activities have very different implications for security prices. A decrease in security creation costs causes the risk-free rate to go up, while a higher appetite for risk-free assets by investors causes the risk-free rate to fall. Given the steady fall in safe yields observed across the world in the past two decades, these intuitive findings confirm the view championed by Bernanke, Bertaut, Demarco, and Kamin (2011), among others, that the recent rise in securitization in the United States has been largely demand-driven.

4Our model assumes no growth in macroeconomic aggregates and should be thought of as a model where all aggregates are de-trended. The prediction for TFP and GDP then is that financial engineering booms can cause both aggregates to fall below trend i.e. bring down their growth rate.
In a final experiment, we consider a demand-driven boom caused by an increase in foreign demand for safe domestic assets. This experiment captures the key features of the Global Saving Glut view associated, for instance, with Bernanke, Bertaut, Demarco, and Kamin (2011). In principle, combining a change in risk appetite with an exogenous inflow of investment funds should lead to bigger output effects than in our closed economy experiments. Once again, the experiment does generate large increases in the volume of cash-flow transformation activities. But it remains the case that much of the resulting increase in spending on the securities created by producers is spent on security creation costs and producer rents rather than capital formation. The output effect is larger than in our other experiments, but much smaller than if the entire foreign inflow of capital flowed to capital formation.

All told then, an increase in foreign demand for safe assets results in a financial engineering boom, a significant decline in safe yields, and an increase in the rents earned by agents that engage in cash-flow transformation, all predictions that accord well with the available evidence. In our model, producers keep and consume their rents but one could trivially introduce intermediaries which source projects, pay producers the value of their outside options, pool and tranche projects as needed, and capture the resulting rents. The resulting model would produce the exact same predictions for aggregate quantities as ours, but would also provide a potential explanation for the vast increase in financial sector rents over the past few decades documented, for instance, by Philippon and Reshef (2012).

These quantitative predictions are, of course, conditional on our modeling assumptions. For instance, we abstract from asymmetric information frictions in the security creation process and do not explicitly model specific changes in the regulatory and tax environment that may have contributed to the recent boom in financial engineering activities. These alternative models may yield bigger effects than those we find. But several aspects of our findings are likely to be robust. First, engineering booms must mean more resources are allocated to security creation and larger rents are earned by the agents who engage in it. While other models may yield different numbers, the wedge between gross investment and capital formation must be present in (and accounted for by) any model of costly security creation. Second,
the fact that capital deepening caused by demand for safe securities should be associated with small, if not negative, effects on TFP stems from the simple idea that projects that only become profitable as a result of cash-flow transformation are unlikely to be high-productivity projects. This economic force is independent of the specific frictions that affect the security creation process. Third, shocks that make security creation cheaper or increase demand for it, unlike shocks that relax borrowing constraints across the board, as common in the traditional financial development literature, do not have any partial equilibrium impact on output. This leaves general equilibrium effects – changes in relative prices – as the only source of impact on macroeconomic aggregates. This, we believe, strongly stacks the deck against finding large effects. Finally, the steady decline in safe yields does constitute strong evidence that demand factors played a dominant role in the recent increase in financial engineering.

Gennaioli, Shleifer, and Vishny (2013) also present a model where more demand for safe assets results in more securitization, more investment and more output when investors have rational expectations. In their model, security creation is free so that expanding financial engineering has no impact on resource use. Their main point, however, is that when investors fail to take into account small probability events (a behavior they term neglected risk, and a violation of rational expectations), the impact of financial engineering booms on output becomes qualitatively ambiguous. These booms do lead to more investment and more output during expansions but, on the other hand, result in greater leverage by financial intermediaries which makes recessions more severe. We find that even when investors have rational expectations, booms in financial engineering are unlikely to be associated with large output gains.

Our paper is also related to, although substantially different from, the growing “too-much-finance” literature that argues that the effect of financial development on growth and productivity becomes weaker, if not negative, at high levels of financial development. Arcand, Berkes, and Panizza (2015), for instance, make the empirical case that once private credit

5See Sahay, Cihak, N’Diaye, Barajas, Pena, Bi, Gao, Kyobe, Nguyen, Saborowski, Svirydzenka, and Yousefi (2015) for a recent review of the empirical literature.
reaches 100% of GDP, additional increases in private intermediation have a negative impact on growth. A common explanation for the tapering that occurs at high development levels is that once the allocative benefits of better credit markets are exhausted, the nature of financial activity expansion changes. Whereas at early stages of development credit expansion leads to the funding of new and highly productive projects, eventually financial development emphasizes security engineering activities. Based for instance on the aforementioned paper by Gennaioli, Shleifer, and Vishny (2013), or classical arguments formalized by, e.g., Tobin (1984) that large financial sectors inefficiently draw skilled human capital away from the production sector, this literature makes the case that too much finance may be detrimental to growth.

While they are consistent with a weak correlation between financial engineering activities and output at high levels of development, the inference one should draw from our findings is quite different from the too-much-finance view. Financial engineering serves a clear, beneficial social role in our framework. When the fraction of risk-averse agents increases, the economy optimally responds by creating more safe assets, even though this is a costly activity. A social planner who must bear the same security creation costs as our producers would respond in the same fashion. As in Allen and Gale (1988) (or in Gennaioli, Shleifer, and Vishny (2013), when investors have rational expectations) our equilibria are constrained-efficient. The points we make in this paper are strictly positive: there is no reason to expect a large positive impact of financial engineering booms on macroeconomic aggregates.

Even though they have a limited effect on aggregate output and productivity, security creation booms do have significant welfare consequences by changing the distribution of financial income and consumption across investor types. Average household welfare increases monotonically as creation costs fall. However, when creating safe securities becomes cheaper, the excess returns earned by investors who are willing to take on more risk fall as does, therefore, their consumption. On the flip side, risk-averse investors become significantly better off. Financial engineering booms triggered by an increase in the fraction of risk-averse agents, on

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6 Philippon and Reshef (2013) make the case that skilled workers in Finance earn excessive rents.
the other hand, strongly benefit risk-tolerant investors and hurt risk-averse investors because the relative demand for risky assets falls. Increased foreign demand for safe securities hurts all domestic investors by lowering all returns but the rents earned by security issuers rise markedly.

2 The environment

Consider an economy in which time is discrete. Each period, a mass one of two-period lived households is born. Each household is endowed with a unit of labor which they deliver inelastically in the first period of their life for a competitively determined wage. There are two types of households – type $A$ and type $N$ – that differ in terms of how they value consumption plans, as we will explain below. Denote the fraction of type $A$ households by $\theta$ while $1 - \theta$ denotes the fraction of type $N$ households born each period.

The economy also contains a large mass of two-period lived producers born at each date $t$. In the first period of their life, each producer can choose to operate a project whose activation requires an investment of one unit of the consumption good at the start of the period. An active project operated by a producer of skill $z_t > 0$ yields gross output

$$z_t^{1-\alpha} n_t^\alpha$$

at the end of period $t$, where $\alpha \in (0, 1)$ and $n_t$ is the quantity of labor employed by the project.

The skill level, $z_t$, of a particular producer is subject to aggregate uncertainty. Producers must decide whether to activate their project before knowing whether aggregate conditions $\eta \in \{B, G\}$ are good ($G$) or bad ($B$). The aggregate shock follows a first-order Markov process with known transition function $T : \{B, G\} \rightarrow \{B, G\}$. Producer types, therefore, are a pair, $z = (z_B, z_G) \in \mathbb{R}^2_+$ of skill levels. A producer of type $(z_B, z_G)$ is of productivity $z_B$ during bad times and $z_G$ during good times. The mass of producers in a given Borel set $Z \subset \mathbb{R}^2_+$ is
\( \mu(Z) \) in each period. We assume that \( \mu \) has continuous derivatives\(^7\) and that producer types are public information.

Producers have linear preferences and can either consume at the beginning of the first period of their life or at the beginning of the second period, although they heavily discount late consumption. Specifically, a consumption profile for producers born at date \( t \) is a triplet \((c_{y,t}^P, c_{o,t+1}^P(B), c_{o,t+1}^P(G))\) where \( c_{y,t}^P \) is their consumption at the start of the first period of their life while \((c_{o,t+1}^P(B), c_{o,t+1}^P(G))\) is their second-period consumption, which may depend on the realization of the aggregate shock at time \( t \). They rank those consumption profiles according to

\[
\epsilon_{y,t} + \epsilon E\left(c_{o,t+1}^P(\eta)|\eta_t\right),
\]

where \( \epsilon \) is a small but positive number.

After the aggregate shock is realized, conditional on having activated a project, and taking the wage rate, \( w_t \), as given, a producer of talent \( z \) chooses her labor input by solving

\[
\Pi(w_t; z) \equiv \max_{n>0} z^{1-\alpha} n^\alpha - nw_t,
\]

where \( \Pi \) denotes net operating income. Let

\[
n^*(w_t; z) \equiv \arg\max_{n>0} z^{1-\alpha} n^\alpha - nw_t
\]

denote the profit-maximizing labor used, given values of the aggregate shock and the wage. We note, for future reference, that \( n^* \) is linear in the realized level \( z \) of skill.

Active producers finance the investment of capital they need by selling securities, i.e. claims to their end-of-period output, to households. Selling one type of security is free, but selling two different types of securities carries a fixed cost \( \zeta > 0 \). One interpretation of this cost is that the agent types are physically separated from one another. Producers must decide

\(^7\)This is for simplicity only. The case where \( \mu \) features positive mass points can be handled by introducing lotteries, as in Halket (2014).
whether to locate near one type or near the other. Delivering payoffs to the closer type is free – this is a mere normalization– delivering payoffs to the more distant type is costly.  

As in Allen and Gale (1988), producers are small hence, when considering which securities to issue, they take as given households’ willingness to pay for marginal investments in the associated payoffs. Formally, let \( q_{N,t}(x_B, x_G) \) be the price at which a marginal amount of a security with payoffs \((x_B, x_G) \geq (0, 0) \) at date \( t \) can be sold to type \( N \) households, where payoffs may depend on aggregate conditions. Similarly, let \( q_{A,t} \) be the price at which contingent securities can be sold to type \( A \) households. Active producers of type \((z_B, z_G)\) choose non-negative security payoffs and consumption profiles to maximize

\[
q_{A,t} (x_{A,t}(B), x_{A,t}(G)) + q_{N,t} (x_{N,t}(B), x_{N,t}(G)) - 1 - \zeta 1_{\{x_{A,t} > 0, x_{N,t} > 0\}} + \epsilon E \left( c_{o,t+1}(\eta)|\eta_{t-1} \right)
\]

subject to:

\[
x_{A,t}(B) + x_{N,t}(B) + c_{o,t+1}^P(B) \leq \Pi(w_t(B); z_B),
\]

\[
x_{A,t}(G) + x_{N,t}(G) + c_{o,t+1}^P(G) \leq \Pi(w_t(G); z_G),
\]

\[
q_{A,t} (x_{A,t}(B), x_{A,t}(G)) + q_{N,t} (x_{N,t}(B), x_{N,t}(G)) \geq 1 + \zeta 1_{\{x_{A,t} > 0, x_{N,t} > 0\}},
\]

where the indicator \( 1_{\{x_{A,t} > 0, x_{N,t} > 0\}} \) takes value one when a non-zero payoff is sold to each household type. The last condition simply says that proceeds from selling securities must cover funding needs at the start of the period. Clearly, producers become active when that constraint can be met since in that case (and only in that case) they enjoy non-negative consumption.

Securities, therefore, are mappings from the aggregate state to a non-negative dividend. Allowing negative dividends would be formally similar to allowing households to short-sell securities. As is well known, doing so can lead to non-existence, even in one-period versions

\[\text{Micro-foundations based on contractual frictions such as limited commitment, asymmetric information or costly verification can also justify this cost structure. We broadly think of } \zeta \text{ as proxying for all costs associated with selling securities in distinct markets or managing a more complex capital structure.}\]
of the environment we describe. More importantly perhaps, financial engineering could not generate private profits if short-sales were unlimited, since any value created by splitting cash-flows could be arbitrated away in the traditional Modigliani-Miller sense.\footnote{See Allen and Gale (1988) for the formal version of this argument.} As a result, no costly security creation would take place in equilibrium.

Producers engage in cash-flow transformation themselves as opposed to delegating that activity to financial intermediaries. One could easily introduce intermediaries that would pool and tranche projects on behalf of producers and distribute cash-flow realizations to households of each type. Since this does not have any impact on equilibrium allocations, we dispense with this modeling layer to simplify the exposition. One area where this choice matters is in the interpretation of producer rents. If intermediaries have the market power to pay producers the value of their outside options (here, zero), they would be the agents consuming the resulting rents.\footnote{As Philippon and Reshef (2012) explain, financial sector rents have increased dramatically as financial engineering activities have boomed.} We will return to this equivalence in section 4.4.\footnote{The fact that production is only subject to aggregate uncertainty implies that combining projects serves no purpose in our model. One could introduce idiosyncratic risk (for instance, projects could be subject to failure) in which case pooling would potentially play a role. However, investors can eliminate this risk on their own so that the resulting model makes identical predictions to ours. This leaves the possibility that investors' ability to diversify is so limited that agents who can pool projects on their behalf serve a quantitatively important purpose but this seems difficult to justify in nations with reasonably developed financial markets.}

Households take as given the set of securities available at the start of a particular period. From their point of view, the menu of securities is a set of gross returns

$$R_{i,t}(z, \eta) = \frac{x_{i,t}(z, \eta)}{q_{i,t}(x_{i,t}(z, B), x_{i,t}(z, G))}$$

on the security issued by producers of type $z = (z_B, z_G) \in \mathbb{R}^2_+$ for household type $i \in \{A, N\}$ with the convention that $R_{i,t}(z) = 0$ if type $z$ is inactive.

Consider a household of type $N$ born at date $t$. They earn wage $w_t$ when young. They consume a part $c_{y,t}^N$ of those earnings and enter the second period of their life with wealth $w_t - c_{y,t}^N$. They allocate that wealth to the securities available at that time by choosing a quantity $a_{i,t}^N(z) \geq 0$ to invest in the securities produced by each producer type $z$. Investment
decisions are made before uncertainty is realized in the final period of their life. At the end of that second period, they consume portfolio proceeds \[
\int a^N_t(z) R_{N,t}(z, \eta) d\mu,
\]
where \(\eta\) is the realization of the aggregate shock. Formally, given \(w_t\), type \(N\) households born at date \(t\) solve:

\[
\max_{a^N_t(z), c^N_{y,t}, c^N_{o,t+1} \geq 0} \log(c^N_{y,t}) + \beta \log \left\{ E \left( c^N_{o,t+1}(\eta) | \eta \right) \right\}
\]

subject to:

\[
c^N_{y,t} = w_t - \int a^N_t(z) d\mu,
\]
\[
c^N_{o,t+1}(B) = \int a^N_t(z) R_{N,t}(z, B) d\mu,
\]
\[
c^N_{o,t+1}(G) = \int a^N_t(z) R_{N,t}(z, G) d\mu,
\]

where \(\beta > 0\).

Given these preferences, type \(N\) households consume a fixed fraction of their earnings in the first period of their life. Once they become old, they have risk-neutral preferences over the remaining consumption plans. As a result, old type \(N\) agents invest all their wealth in those securities whose expected return is highest. Therefore, letting

\[
\bar{R}_{N,t} = \max_z T(B|\eta_{t-1}) R_{N,t}(z, B) + T(G|\eta_{t-1}) R_{N,t}(z, G),
\]

old risk-neutral agents are willing to pay:

\[
q_{N,t}(x(B), x(G)) = \frac{T(B|\eta_{t-1}) x(B) + T(G|\eta_{t-1}) x(G)}{\bar{R}_{N,t}}
\]

for a marginal investment in a security with payoff \((x(B), x(G))\) at date \(t\).

Similarly, type \(A\) agents born at date \(t\) solve

\[
\max_{a^A_t(z), c^A_{y,t}, c^A_{o,t+1} \geq 0} \log(c^A_{y,t}) + \beta \log \left\{ \min \left\{ c^A_{o,t+1}(B), c^A_{o,t+1}(G) \right\} \right\}
\]
subject to:

\[ c^A_{y,t} = w_t - \int a^A_t(z) d\mu, \]

\[ c^A_{o,t+1}(B) = \int a^A_t(z) R_{A,t}(z, B) d\mu, \]

\[ c^A_{o,t+1}(G) = \int a^A_t(z) R_{A,t}(z, G) d\mu. \]

Old agents of type A, in other words, are infinitely risk-averse and try to maximize the value of worst-case scenario consumption. Their preferences are also such that they save a fixed fraction of their earnings when young.

Consider an old household of type A alive at date \( t \). Define

\[ \bar{R}_{A,t} = \min \left\{ \frac{c^A_{o,t}(B)}{a^{i,-1}_t}, \frac{c^A_{o,t}(G)}{a^{i,-1}_t} \right\} \]

as the effective return these agents realize on their investment at the optimal solution to their problem. If \( c^A_{o,t}(B) < c^A_{o,t}(G) \) at the optimal solution, their willingness to pay for a marginal investment in a security with payoffs \((x(B), x(G))\) is

\[ q_{A,t}(x(B), x(G)) = \frac{x(B)}{\bar{R}_{A,t}}. \]

Indeed, they only value marginal payoffs in the lowest consumption state in that case. The symmetric property must hold when \( c^A_{o,t}(B) > c^A_{o,t}(G) \). When \( c^A_{o,t}(B) = c^A_{o,t}(G) \), which we will soon argue must hold in equilibrium at all dates,

\[ q_{A,t}(x(B), x(G)) = \frac{\min(x(B), x(G))}{\bar{R}_{A,t}}. \]

Having stated every agent’s optimization problem, we can now define an equilibrium. Old households of type \( i \in \{A, N\} \) enter date 0 with wealth \( a_{i,-1} > 0 \). The aggregate state of the economy at date 0 is fully described by \( \Theta_0 = \{a_{A,-1}, a_{N,-1}, \eta_{-1}\} \) where \( \eta_{-1} \in \)
It is the aggregate shock at date \( t = -1 \). An equilibrium, then, is a list of security payoffs \( \{ x_{i,t}(z, \eta) \} \) for each household type, producer type and aggregate shock, the associated returns \( \{ R_{i,t}(z, \eta) \} \), consumption profiles \( \{ c_{y,t}^P, c_{o,t+1}^P(B), c_{o,t+1}(G) \} \) for each producer type and a corresponding set \( Z_t \) of active producers, wage rates \( \{ w_t(\eta) \} \) for each \( \eta \in \{ B, G \} \), consumption plans and security purchases \( \{ c_{i,t}^P, c_{o,t+1}^P(B), c_{o,t+1}(G), a^i_t(z) \} \) for each household type and, finally, pricing kernels \( \{ q_{A,t}, q_{N,t} \} \) such that, at all dates and for all possible histories of aggregate shocks:

1. Security purchases and consumption plans solve the households’ problem;

2. Security menus and consumption plans solve each producer’s problem;

3. The market for labor clears:

\[
\int_{Z_t} n^*(w_t(\eta); z) d\mu = 1 \text{ for } \eta \in \{ B, G \};
\]

4. The market for each security type clears\(^{12}\):

\[
\int_{Z_t} \theta a^A_t(z) R_{A,t}(z, \eta) d\mu = \int_{Z_t} x_{A,t}(z, \eta) d\mu
\]

\[
\int_{Z_t} (1 - \theta) a^N_t(z) R_{N,t}(z, \eta) d\mu = \int_{Z_t} x_{N,t}(z, \eta) d\mu
\]

for \( \eta \in \{ B, G \} \);

5. Pricing kernels are consistent with the household’s willingness to pay for marginal payoffs, i.e.:

\[ q_{N,t}(x(B), x(G)) = \frac{T(B|\eta_{t-1})x(B) + T(G|\eta_{t-1})x(G)}{R_{N,t}}, \]

\[ q_{A,t}(x(B), x(G)) = \min(x(B), x(G)) \frac{x(B)}{R_{A,t}} \text{ if } c_B^A(t)(B) = c_B^A(t)(G), \]

\(^{12}\)For simplicity we only state an aggregate market clearing condition for each household type. This is without loss of generality since in equilibrium agents are exactly willing to hold each producer’s securities. Equivalently, securities of each type can be pooled at no cost.
(c) $q_{A,t}(x(B), x(G)) = \frac{x(G)}{R_{A,t}}$ if $c_{o,t}^A(B) > c_{o,t}^A(G)$,
(d) $q_{A,t}(x(B), x(G)) = \frac{x(B)}{R_{A,t}}$ if $c_{o,t}^A(B) < c_{o,t}^A(G)$,

for all possible securities $(x(B), x(G)) \geq (0, 0)$ where:

$$\bar{R}_{N,t} = \max_z T(B|\eta_{t-1}) R_{N,t}(z, B) + T(G|\eta_{t-1}) R_{N,t}(z, G),$$

while

$$\bar{R}_{A,t} = \frac{\min\{c_{o,t}^A(B), c_{o,t}^A(G)\}}{a_{t-1}}.$$

The final equilibrium condition is similar to the consistency condition imposed by Allen and Gale (1988). Because type A households have Leontieff preferences, we cannot simply write, as they do, that pricing kernels are marginal rates of substitutions but the economic content of the condition is exactly the same. Producers take pricing kernels as given and choose securities to maximize their profits. Consumers, given this menu of securities, choose an optimal consumption plan which implies their marginal willingness to pay of securities. The implied kernels have to coincide with the kernels assumed by producers.

### 3 Properties of equilibria

The state of the economy at the start of a period is fully described by the wealth of old households $a_{i,t-1} > 0$ for $i \in \{A, N\}$ and the most recent aggregate shock $\eta_{t-1}$. For every possible value of these three objects we need to find pricing kernels $(q_{A,t}, q_{N,t})$ as well as wage rates $(w_t(B), w_t(G))$ for each possible realization of the aggregate shock, such that all markets clear and the Allen-Gale condition (equilibrium condition 5) is satisfied. This is a static problem which we characterize in this section. Since households simply save a fixed fraction of their wages in each period, a simple law of motion will then fully describe an equilibrium.
3.1 Security space

The following result greatly simplifies the analysis.

**Lemma 1.** In any equilibrium, the consumption of risk-averse agents is risk-free and they only purchase risk-free securities. Furthermore, in any equilibrium,

\[ \bar{R}_{N,t} \geq \bar{R}_{A,t} \]

with a strict inequality whenever \( \zeta > 0 \) and a positive mass of producers issue two securities.

*Proof.* Assume, by way of contradiction, that an equilibrium exists in which, in a given period, the consumption bundle \((c_B, c_G)\) of old risk-averse agents is such that \(c_B > c_G\). Then, given their preferences, risk-averse agents pay nothing for the bad-realization payoff on any security, as their marginal valuation of consumption in bad times is zero. Moreover, in order for \(c_B > c_G\) to hold, a positive mass of securities with higher payoffs in the bad state than in the good state must be sold to risk-averse agents. But those producers would be strictly better off either selling the bad state payoff to risk-neutral agents, or simply consuming it themselves. The case in which \(c_B < c_G\) can be similarly ruled out.

To see why risk-neutral agents must earn a premium assume that the opposite holds. Then producers would earn strictly more on any security sold to risk-neutral agents. But this would contradict the fact that the supply of securities to risk-averse investors must be strictly positive, since they always have strictly positive wealth. Finally, if producers bear the cost in order to sell two securities, the benefit of doing so, compared to selling everything to risk-neutral agents, must be strictly positive. \(\square\)

Given this result, it must be that in any equilibrium

\[ q_{A,t}(x(B), x(G)) = \frac{\min(x(B), x(G))}{\bar{R}_{A,t}} \]
where

\[ \hat{R}_{A,t} = \min_{\Pi(B,z)} \left\{ c_A^A(B), c_{o,t}^A(G) \right\} \]

Furthermore, since it only makes sense to issue risk-free securities to risk-averse agents, producers choose a risk-free payoff \( x_A \geq 0 \), risky-payoffs \( x_N \) for type \( N \) agents, and an end of period consumption plan \( c_o^p \) to maximize:

\[
\frac{x_A}{\hat{R}_{A,t}} + \frac{T(G|\eta_{t-1})x_N(G) + T(B|\eta_{t-1})x_N(B)}{\hat{R}_{N,t}} - 1 - \zeta_1 \mathbb{1}_{\{x_A>0 \text{ and } x_N>0\}} + \epsilon E(c_o^p|\eta_{t-1}),
\]

where feasibility, i.e., the non-negativity restriction on security payoffs imposes:

\[
x_A \leq \min \{ \Pi(w(B);z_B), \Pi(w(G);z_G) \}
\]

\[
x_A + x_N(B) + c_o^p(B) \leq \Pi(w(B);z_B),
\]

\[
x_A + x_N(G) + c_o^p(G) \leq \Pi(w(G);z_G).
\]

The first restriction says that risk-free payoffs must indeed be risk-free and hence have to be deliverable even under the worst-case realization of profits. The other two restrictions are feasibility conditions for each possible realization of the aggregate state.\(^{13}\)

To ease notation in the statement of our next result, write

\[ \Pi(z) = \min \{ \Pi(w(B);z_B), \Pi(w(G);z_G) \} \]

\(^{13}\)In light of this result, our model appears equivalent to traditional models of corporate finance where producers choose a combination of debt and equity to maximize the overall value of the project. But it is in fact fundamentally different from those traditional models, in the same sense that Allen and Gale (1988) is fundamentally different from those models, for several reasons. Traditional models of corporate finance (trade off between the tax advantages of debt and its consequences of manager incentives, pecking order, etc.) can be fully cast in environments with a representative investor or one exogenous pricing kernel. Heterogeneity plays no role whatsoever in those approaches. Our model, in contrast, relies on designing securities with features that appeal to investors with different tastes, which is arguably the driving force behind the recent increase in financial engineering. Furthermore, the key distinction in our model between the two broad types of securities it generates is that one type is safe whereas the other is risky. Our model thus maps to highly rated securities vs other securities, not to debt vs equity. Again, this seems appropriate because the primary outcome of securitization activities is to create highly rated securities collateralized by risky assets.
as short-hand notation for the lowest possible realization of profits for a particular producer at a particular history, and denote the state where the lowest profit is realized as $\eta(z)$. By the same token, let

$$\bar{\Pi}(z) = \max\{\Pi(w(B); z_B), \Pi(w(G); z_G)\}$$

be short-hand for the highest possible realization of profits, and $\bar{\eta}(z)$ denote the state where the highest possible profit is realized.

The following proposition states that the solution to the producer problem satisfies a simple bang-bang property. Producers that tranche cash flows and issue two types of securities sell as much risk-free securities as possible.

**Proposition 2.** In an equilibrium where a positive mass of producers pays the security creation cost $\zeta$, either $x_A(z) = 0$ or $x_A(z) = \Pi(z)$ for $\mu$-almost all producer types $z$.

**Proof.** Consider a producer that paid creation cost $\zeta$ in a particular period. In light of lemma 1, any solution to her security creation problem must involve $x_A > 0$. Consider any feasible choice $(x_A, x_N, c_o^P)$ such that $x_A > 0$ but $x_A < \Pi(z)$. Then, a slight increase in $x_A$ would increase the producer’s objective by

$$\frac{1}{R_{A,t}} - \max\left\{\epsilon, \frac{T(G|\eta_{t-1}) + T(B|\eta_{t-1})}{R_{N,t}}\right\} > 0.$$

Indeed, lemma 1 guarantees the inequality with respect to the second element of the max operator. Moreover, it must also be the case that $\frac{1}{R_{A,t}} > \epsilon$ (and that $\frac{1}{R_{N,t}} > \epsilon$ for that matter) since otherwise it would not make sense to pay the security creation cost in the first place, as the producer could simply sell one type of securities and consume the remainder. The result follows.

This result is a consequence of a fundamental feature of environments in the spirit of Allen and Gale (1988) such as ours: producers take state prices as given, hence have a linear objective defined over a convex set, which, leads to bang-bang financial policies. This has
nothing to do with the fact that our investors are either fully risk-neutral or fully risk-averse. Producer problems solve a linear problem simply because they are small, hence their actions have no impact on pricing kernels. When producers choose to create some risk-free debt, they maximize the production of such debt.

When is it profitable for producers to engage in costly security creation? Recall from lemma 1 that $\bar{R}_{N,t} > \bar{R}_{A,t}$ so that producers earn strictly more gross revenues by selling to both agent types rather than simply dealing with risk-neutral agents. That gain in revenue must exceed the fixed cost $\zeta$. Their expected profit net of capital costs is

$$\frac{T(\bar{\eta}(z)|\eta_{t-1}) (\bar{\Pi}(z) - \Pi(z))}{\bar{R}_{N,t}} + \frac{\Pi(z)}{\bar{R}_{A,t}} - \zeta,$$

while a producer that sells exclusively to risk-neutral agents has expected profit net of capital costs of

$$\frac{T(\bar{\eta}(z)|\eta_{t-1})\bar{\Pi}(z) + T(\eta(z)|\eta_{t-1})\Pi(z)}{\bar{R}_{N,t}},$$

which implies that a producer will prefer to issue two securities to just catering to risk-neutral agents when $\Pi(z) \left( \frac{1}{\bar{R}_{A,t}} - \frac{1}{\bar{R}_{N,t}} \right) \geq \zeta$. This happens when the security creation cost is sufficiently low, when the difference between the returns paid to the two types is large enough, and importantly, when the worst possible profit is large enough. In particular, the decision between tranching cash flows or issuing risky securities exclusively does not depend on the highest possible profits $\Pi(z)$.

Issuing two security types must also dominate issuing riskless assets only. When a producer of type $z$ only issues risk-free assets, her utility is

$$\frac{\Pi(z)}{\bar{R}_{A,t}} + \epsilon (\bar{\Pi}(z) - \Pi(z)).$$

Issuing both types of securities is preferable when

$$\left( \frac{T(\bar{\eta}(z)|\eta_{t-1})}{\bar{R}_{N,t}} - \epsilon \right) (\bar{\Pi}(z) - \Pi(z)) \geq \zeta.$$
Intuitively, the producers that issue safe securities only are those whose expected profits are sufficiently similar across states.\textsuperscript{14} These considerations will play a key role in interpreting the outcome of our upcoming simulations.

\subsection*{3.2 Aggregation and GDP accounting}

The aggregate production function that results from adding up the individual projects’ production plans takes a familiar neoclassical form. In order to derive it, let \( Z_\Theta \subseteq R^+_2 \) denote the set of types that operate projects (an equilibrium object) given the aggregate state \( \Theta \), where we dispense with time subscripts to reduce clutter. The endogenous set \( Z_\Theta \) plays a key role in many of our upcoming results since it captures all producer selection effects but, for the purpose of aggregation, it is enough to take it as given.

Let \( K \) denote the aggregate quantity of capital used to operate active projects in a given period. In equilibrium, this has to equal the measure of projects activated, as each project requires one unit of the consumption good to be activated:

\[
K = \int_{Z_\Theta} d\mu.
\]

It will be useful to define the average productivity among active projects when the realization of the aggregate state is \( \eta \in \{B, G\} \):

\[
\bar{z}(\eta) \equiv \frac{\int_{Z_\Theta} z_\eta d\mu}{\int_{Z_\Theta} d\mu},
\]

and to note that this implies \( K \bar{z}(\eta) = \int_{Z_\Theta} z_\eta d\mu \).

In equilibrium, the measure of labor supplied is one at all dates, but generalizing to other, off-equilibrium employment levels, let \( N \) denote the total mass of employment. Then, for the labor market to clear, and using the solution to the projects’ labor choice problem, we must

\textsuperscript{14}In our upcoming simulations, this is illustrated by the diagonal swaths on the top two panels of Figure 2.
have that for each possible realization, \( \eta \), of the aggregate shock:

\[
N = \int_{Z_{\theta}} n^*(z_{\eta}, w(\eta))d\mu
= n^*(1, w(\eta)) \int_{Z_{\theta}} z_{\eta}d\mu
= n^*(1, w(\eta))K \tilde{z}(\eta).
\]

We can now write the aggregate production function given aggregate capital, aggregate labor and the aggregate productivity shock:

\[
F(\eta, K, N) = \int_{Z_{\theta}} z^{1-\alpha}_{\eta} n^*(z_{\eta}, w)^\alpha d\mu
= \int_{Z_{\theta}} z_{\eta}n^*(1, w(\eta))^\alpha d\mu
= \int_{Z_{\theta}} z_{\eta} \left( \frac{N}{K \tilde{z}(\eta)} \right)^\alpha d\mu
= \left( \frac{N}{K \tilde{z}(\eta)} \right)^\alpha \int_{Z_{\theta}} z_{\eta}d\mu
= \tilde{z}(\eta)^{1-\alpha} K^{1-\alpha} N^\alpha. \tag{3.1}
\]

This is a standard-looking neoclassical production function, where the endogenous term \( \tilde{z}(\eta)^{1-\alpha} \) plays the role of measured TFP, which in this environment is a function of the average efficiency of activated projects.

As we will discuss in more depth in section 4, this expression immediately implies that the effects of security creation booms on TFP must be ambiguous. Unlike in traditional models of financial development, there are no untapped efficiency gains at the project level. The net impact of any change in the environment on TFP boils down to whether new entrants are more or less productive than already active and exiting producers. If anything, and as we will confirm via numerical simulations later, new entrants following a drop in security creation costs are more likely to be relatively low-productivity producers. Simply put, highly
productive producers are active regardless of whether security creation is cheap or expensive.

The set of equilibrium conditions defined above implies an aggregate feasibility constraint that must hold every period. On the expenditure side, define aggregate consumption as the sum of each agent type’s consumption,

\[ C_t \equiv \theta (c_{A,t} + c_{A,t}) + (1 - \theta) (c_{N,t} + c_{N,t}) + c_{o,t} + c_{y,t} \]

where \( c_{o,t} \) is the second-period consumption of producers born at date \( t - 1 \) while \( c_{y,t} \) is the first period consumption of producers born at date \( t \).

Aggregate investment is the sum of next period’s capital and the expenditures intermediaries incur in creating new securities:

\[ I_t = K_{t+1} + \int_{Z_{o}} \zeta 1_{\{x_A > 0 \text{ and } x_N > 0\}} d\mu. \]

The result is that we can express the aggregate feasibility constraint in a familiar form,

\[ C_t + I_t = Y_t. \]

That is, GDP equals the sum of aggregate consumption and investment.

### 3.3 Existence and Comparative Statics

In a given period and in any current state – a triplet \( \{a^A, a^N, \eta_{-1}\} \) of wealth for the two household types and aggregate shock in the most recent period – existence of an equilibrium will be guaranteed if rates of return \((\bar{R}_A, \bar{R}_N)\) and wages \(w(B), w(G)\) can be found so that security and labor markets clear. The associated law of motion for wealth simply follows from the fact that wealth levels at the start of a given period are the previous period’s wage, times the fixed savings rate implied by the preferences we have specified. Existence thus boils down to solving a classical fixed point problem with four distinct markets.
Proposition 3. *An equilibrium exists.*

*Proof.* See appendix.

Having established the existence of equilibria, we can now use our framework to study the relationship between the intensity of financial engineering activities and macroeconomic aggregates. There are at least two natural measures of the quantity of financial engineering in our environment. First, we can measure the volume of securities issued by producers who choose to bear the security creation cost. Second, we can measure the amount

$$
\int_{Z_\Theta} \zeta 1_{\{x_A > 0 \text{ and } x_N > 0\}} d\mu
$$

producers spend on security creation activities. Both measures are, of course, endogenous. We will consider two changes to the environment that may cause these equilibrium quantities to rise.

First and perhaps most naturally, a drop in the security creation cost $\zeta$, holding prices constant, can only cause an increase in the fraction of producers who choose to bear that cost. Whether that drop also causes a decline in aggregate security creation costs is ambiguous since, while the cost per producer is cheaper, more producers choose to bear it. Second, we will consider a permanent increase in the fraction of risk-averse agents. We view this second experiment as proxying for the well-documented increase in appetite for safe assets over the past two decades. This second experiment potentially makes the extraction of safe securities from risky projects more profitable. Loosely speaking, our first exercise corresponds to a supply-driven increase in engineering activities while the second corresponds to a demand-driven increase.

Tracing the effects of these shocks is greatly complicated by the fact that they both have an impact on prices in general equilibrium. For instance, we would expect greater demand for safe assets to depress safe returns. In the next section, we resort to calibrated numerical simulations to quantify the effects of these shocks.

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In this section, we preview the results one should expect from these quantitative explorations using a parametric example. Assume that producers are scaled up versions of one another in the sense that $\frac{z_G}{z_B}$ is $\mu$-almost surely a constant. Put another way, almost surely, $z_G = zA_G$ while $z_B = zA_B$, where $z > 0$ is the producer’s skill level and $A_G > A_B > 0$ are aggregate shocks common to all producers. Under those assumptions, the search for market clearing wages becomes one dimensional. The fact that $Z_\Theta$ is set prior to the realization of the aggregate shock, and hence is the same regardless of that realization, also means that if we know what bad time wages $w(B)$ are in a particular period, only one value of $w(G)$ can also clear the labor market during good times. Furthermore, the Cobb-Douglas functional forms we have assumed for production functions imply that $\frac{w(G)}{w(B)}$ is a constant greater than one. In this case, producer talent is summarized by a scalar $z \in \mathbb{R}_+$. Security creation policies, as a result, become simple.

**Lemma 4.** Assume that $\frac{z_G}{z_B}$ is $\mu$-almost surely a constant. Then security creation policies are fully characterized by two thresholds $z_t \leq \bar{z}_t$ in every period. Producers become active when $z_B > z_t$ and bear the security creation cost when $z_B > \bar{z}_t$.

The intuition for this result is simple. Only producers whose scale is high enough can generate enough security creation profits to overcome the fixed cost. Since in this parametric example producer types are one-dimensional, only the most qualified producers choose to create different securities for each type.

We can further simplify the example by assuming that, holding other parameters the same, $\frac{z_G}{z_B}$ is high enough that it is never profitable for any producer to only sell securities to risk-neutral agents. Given $\zeta$, when $\frac{z_G}{z_B}$ is high enough, the gap in profits between good and bad times is so high that producers are always better off selling the excess profits they generate in good times to risk-neutral agents rather than consuming it. With this assumption, producers whose talent is between the two thresholds described in the lemma sell their entire output to risk-neutral agents.

So consider now a marginal drop in security creation costs $\zeta$ in a particular period. Holding
prices the same, the drop cannot have any effect on the lower threshold since, at that threshold, producers only issue one security. In turn, and once again holding prices the same, wages, output and aggregate TFP cannot change. It follows that, in this example, general equilibrium effects are the only possible source of impact of drops in $\zeta$ on macroeconomic aggregates. At the original prices and original thresholds, labor markets continue to clear but there is an excess supply of safe securities. So one would expect the risk-free rate to go up and the upper threshold to fall. These, in turn should cause an excess demand for risky securities, which causes the return earned by risk-neutral agents to fall. Holding wages the same, a fall in the return producers have to pay risk-neutral agents causes a fall in the lower threshold, and in turn an increase in labor demand and output.

The fact that changes in security creation costs have no direct, partial equilibrium, effect on producer participation suggests that, quantitatively, their effect on output is bound to be small. Our upcoming numerical simulations will confirm this intuition. More surprisingly, they will show that the effects of lowering security creation costs can be negative. To understand why this can happen, observe that in our model, we must have, following any change in the environment,

$$\text{Change in capital formation} = \text{Change in spending on securities} - \text{Change in security creation expenditures} - \text{Change in producer consumption/rents}.$$ 

We argued above that the first term on the right-hand-side must go up (at least in partial equilibrium) as $\zeta$ falls. Security creation expenditures, on the other hand, cannot be monotonic in $\zeta$ since they are zero when $\zeta$ is zero and must return to zero once $\zeta$ is so large that no cash-flow splitting takes place. There must be regions, in other words, where expenditures on creation costs rise as $\zeta$ falls. Our simulations will show that this effect can be large enough to dominate the behavior of the other components of capital formation. Our simulations will also show that the final term, producer rents (which equal the sum of all security issuance
revenues net of the capital put in place and any security creation costs), can be non-monotonic in \( \zeta \) as well.

4 Numerical simulations

To investigate how the consequences of security creation booms for macroeconomic aggregates may vary depending on what is driving the rise, we run two types of experiments. First, we compare economies that differ only in security creation costs. Starting with an economy with no security creation costs, we increase these costs until no cash-flow splitting takes place in equilibrium. Second, we compare economies that differ only in the fraction of agents of each type (risk-neutral or infinitely risk-averse). In particular, in the second experiment, all economies being compared feature the same security creation cost. Finally, we will consider a version of this demand-side experiment in which the boom in demand for safe assets comes from foreign investors.

4.1 Parameterization and algorithm

In our model economy, agents live for two periods. We will therefore think of a period as representing 25 years. We set the elements of the aggregate state’s transition matrix \( T \) so that the probability of remaining in the bad state is \( T_{BB} = 0.2 \) and the probability of remaining in the good state is \( T_{GG} = 0.9 \). This implies that the economy spends close to 90 percent of the time in the good state. Together with the assumed length of a given period, this means that one should think of the bad state as a rare but protracted event. Correspondingly, we will interpret a bad shock as a disaster in the sense of Barro and Ursua (2008) or Gourio (2013).

We set \( \alpha = 0.6 \) which is slightly lower than the typical two-thirds to account for the fact that some of the profits going to entrepreneurs are rewarding labor. We set \( \theta = 0.5 \) in the benchmark implying an equal share of risk-neutral and risk-averse agents. In one of our main experiments, we will vary this share over the entire unit interval. We make \( \epsilon = 0 \) so that
producers fully discount old period consumption. We interpret this value as vanishingly small in the sense that ties between consuming left-over output and selling it for nothing are broken in favor of the first option.

We set the support of project productivities to $\mathcal{Z} = [0, 1] \times [0, 1]$, and assume that $\mu$ follows a truncated bivariate normal distribution with mean $\bar{\mu} = (\bar{z}_G, \bar{z}_B)$ and variance-covariance matrix

$$\Phi = \begin{pmatrix} (\varsigma \bar{z}_G)^2 & 0 \\ 0 & (\varsigma \bar{z}_B)^2 \end{pmatrix}$$

where $\varsigma > 0$. That is, we assume that the two skill levels are uncorrelated at the population level and normalize the two variance terms so that the coefficient of variation of skill is approximately the same in the two aggregate states. Finally, we normalize $\bar{z}_G = 1$ leaving us with three parameters to calibrate: $\bar{z}_G$, $\varsigma$, and the household discount factor $\beta$. We choose these parameters so that, in stochastic steady-state and on average, 1) the risk-free rate is approximately 2%, 2) output in bad times is 15% below output in good times, which is the value Barro and Ursua (2008) use as a cut-off for their empirical definition of a disaster, and 3) the ratio of producer rents to output is 10%, which matches the approximation for this moment obtained in a similar environment by Corbae and Quintin (2016) using US private corporate sector data. The resulting parameter values are $\beta = 0.68$, $\bar{z}_G = 0.11$, and $\varsigma = 0.8$. In our sensitivity analysis, we will consider large variations in these values to gauge the robustness of our key results.

Standard arguments show that our economies eventually converge to a stochastic steady-state, i.e. an invariant distribution of all endogenous variables in our model. To obtain statistics for all endogenous variables in this stochastic steady-state, we adopt a traditional Markov chain Monte Carlo approach. Specifically, our algorithm is as follows:

1. Given parameters, solve for household and intermediary policy functions for every pos-

\footnotetext[15]{Because of the truncation the two coefficient of variations are not exactly the same.}
\footnotetext[16]{See Brock and Mirman (1972).}
\footnotetext[17]{See Tierney (1994).}
sible aggregate state of the economy;

2. Draw a 100-period sequence of aggregate shocks \( \{\eta_t\}_{t=1}^{100} \) using the Markov transition matrix \( T \) and record the value of all endogenous variables starting from an arbitrary value of aggregate wealth;

3. After dropping the first 10 periods, so that the assumed initial conditions have at most a negligible effect on the value of endogenous variables, compute average values for all endogenous variables.

To facilitate comparisons across economies with different costs, we use the same draw of random aggregate shocks throughout our simulations. Our model features quick transitions to steady-state and we have found that 100 periods suffice to generate stable estimates of the desired moments.

### 4.2 Varying security creation costs

Figure 2 displays producer policies for four different securitization cost levels. A mass of projects is left inactive because they are unprofitable in expected value terms, regardless of the security structure used to finance them. For any given productivity level in the bad state \( z_B \), there is a threshold level of productivity in the good state \( \bar{z}_G(z_B) \) above which the expected profits cover the cost of capital and any possible security creation costs and, as a consequence, the project is activated. The threshold \( \bar{z}_G(z_B) \) is weakly decreasing in \( z_B \). As \( z_B \) falls, producers, regardless of how they finance their activities, need to be (at least weakly) compensated by increases in \( z_G \).

When security creation costs are zero, issuing risk-free securities is weakly dominated by issuing both types of securities (producers that have exactly the same profits in both states are indifferent between the two). Making costs slightly positive, as we do for \( \zeta = 0.005 \), reveals exactly who these producers are, as issuing risk-free securities only becomes slightly more profitable than tranching and, as a consequence, a sliver of active producers starts to
do so as shown in panel A of Figure 2. As costs increase further, the measure of producers who issue only risk-free securities expands around the ray where profits are the same in both states.

Producers who choose to engage in costly security creation have two characteristics. First, they must be productive, hence large enough to justify bearing the fixed cost $\zeta$. Producers whose $z_B$ is low enough and producers whose $z_G$ is low enough choose instead to sell risky securities only. Second, the gap between their profits in the two states must be high enough, since otherwise they would be better off selling risk-free securities only. This yields the two tranching regions in panels A, B, and C of Figure 2. In panel D, security creation costs are so high that no producer type engages in it.

As security creation costs increase, the share of tranching establishments falls monotonically, as shown in panel B of Figure 3. This puts pressure on the relative issuance of risky securities to increase, which, because the demand for the two security types is constant, means that for markets to clear, the price of risk-free securities needs to rise, leading to a corresponding fall in the risk-free rate. The same reason explains why risk-neutral agents pay less for risky securities as security creation costs increase, and therefore enjoy a higher rate of return. Both rates of return are shown in panel C of Figure 3.

To understand what happens to capital formation as costs vary, it is instructive to first look at what happens to expenditures in securitization activities. When $\zeta = 0$, these expenditures are trivially zero. As $\zeta$ increases, they start rising, but eventually fall back down to zero, as the share of tranching establishments goes to zero. This results in a Laffer-curve-like relationship between security creation costs and expenditures, shown in panel D of Figure 3.

Given our assumption that one unit of capital is needed to operate a project, capital formation equals the share of active projects and is given by total spending on securities net of producer rents and security creation expenditures. Because spending on securities is proportional to output (as it is a fixed fraction of wages that are linear in output) and security creation expenditures initially increase, capital formation falls as security creation costs start increasing, as shown in panel A of Figure 3. Eventually, as costs continue to increase but
securitization expenditures start falling, capital formation comes back up, resulting in a non-monotonic relationship with security creation costs. Producer rents are also non-monotonic (see panel E of Figure 4), but they are quantitatively less meaningful.

Output displays the same non-monotonic relationship with securitization costs as capital, as shown in panel A of figure 4. Quantitatively, there is not much difference between the changes in these two variables, resulting in little action in TFP, as shown in panel B of Figure 4. To understand why this is the case, recall the intuition from the one-dimensional example in section 3.3 arguing that the share of operating projects is not affected directly by changes to $\zeta$. It turns out this generalizes to our two-dimensional environment. As Figure 2 illustrates, tranching establishments are not marginal in the sense that they are not close to inactivity. Therefore, the operation threshold only moves because of general equilibrium effects, which are quantitatively small. Since TFP is simply the average productivity of active producers, this results in small TFP changes.

This quantitative irrelevance result is robust to a variety of parameter changes. In the left panel of Figure 15 we show the output that results from re-running the experiment with significantly different skill distributions: one where we halve the standard deviation in the main diagonal of the variance-covariance matrix regulating the skill distribution, and another with double the standard deviation. Although this leads to very different levels of output (which in the panel is indexed to output in the benchmark economy under the lowest security creation cost), it continues to be true that output levels do not vary much with changes in security creation costs. We also experimented with a variety of non-zero elements off the main diagonal, and obtained similar results (not shown).

Since the economy suffers a bad shock once in roughly every 10 periods and each period represents 25 years, a fall in output that is calibrated to 15 percent may seem small. Gourio (2013) uses the same 15 percent, but the probability of disaster in his model economy is 2 percent a year, much higher than in our calibrated model. To show that this does not affect our main results, we recalibrate the mean skill level in bad times to yield a 25 percent difference in output relative to good times. The resulting output is shown in the right panel
of Figure 15. Again, other than the obvious fact that output is on average lower, this does not change the result that the main macroeconomic aggregates do not vary much with even large changes in security creation costs.\footnote{Our results are also robust to changes in the saving rate induced by changes in the discount factor $\beta$. Economies with larger $\beta$’s save more and are therefore richer, but it remains the case that changes in security creation costs have a small impact on capital formation and output.}

4.3 Varying the share of risk-averse investors

In the second experiment we keep the security creation cost fixed at the intermediate level of $\zeta = 0.1$ and vary, instead, the share of the two types of agents, from $\theta = 0.1$ (10 percent risk-averse agents) to $\theta = 0.9$. As the share of risk-averse agent rises, the share of wealth held by those agents increases and so, therefore, does the demand for riskless assets. This puts downward pressure on the risk-free rate and upward pressure on the risky rate, as panel C in Figure 6 shows. As a result, there is a compositional change in the type of active producers: the fraction of establishments financed exclusively by risky securities drops monotonically, as the share of those issuing risk-free securities, whether exclusively or by tranching, increases (panel B of Figure 6).

At the same time, as shown in panel A of Figure 6, the overall share of active producers or, equivalently, capital formation, falls. Initially (compare panels A and B of Figure 5), this decrease is small: as $\bar{R}_N$ increases and the revenues of the marginal producers exclusively issuing risky securities fall, some switch to issuing riskless securities and some exit. Moreover, some hitherto infra-marginal (inactive) projects become profitable by issuing risk-free securities exclusively, as $\bar{R}_A$ drops. As $\theta$ continues to increase (compare panels C and D of Figure 5), market clearing eventually requires large increases in the equilibrium risky rate of return $\bar{R}_A$, driving additional risky producers out, while other producers start tranching, or financing themselves through the exclusive issuance of risk-free securities. All the while, the fall in the share of active producers brings down the demand for labor and the wage rate (as shown in panel C of Figure 7). As the share of active projects fall, so does output, but because
it is the relatively less productive producers who exit as \( \theta \) rises, TFP increases. Again, this increase is quantitatively small.

Unlike what happened in our previous experiment, the consequences of the financial engineering boom for capital formation are not mitigated by the fact that per producer cost is reduced. With fixed per-producer tranching costs, as more producers optimally decide to pay the cost, total security creation expenditures rise monotonically to reach almost 1% of GDP, as shown in panel D of Figure 6.

### 4.4 The global saving glut

The global saving glut view associated for instance with Bernanke, Bertaut, Demarco, and Kamin (2011) attributes the recent increase of financial engineering activities to an increase in foreign appetite for safe US assets. This section carries out an experiment that captures the key features of this phenomenon and describes its consequences in the context of our model. We do so by introducing in our model foreign investors who inelastically demand risk-free assets and by assuming that their demand amounts to a fixed fraction \( \gamma \) of domestic demand.\(^{19}\) The fundamental difference between this experiment and the experiment in which we simply varied the share of risk-averse agents is that foreign participation increases gross investment above national savings, and does so by significant amounts when \( \gamma \) is high.

We initially set the level of security creation costs to a value around which the total expenditure in security creation is maximized (\( \zeta = 0.1 \)) in the benchmark economy with no foreign savings: see panel D of Figure 3. Note that this level of \( \zeta \) is an intermediate one given the Laffer-like relationship between security creation costs and total expenditure. But we will also consider vastly different levels of security creation costs to make sure that our main results are not sensitive to this approach.

Panel A of Figure 10 shows that, unsurprisingly, as more foreign capital flows into the

\(^{19}\)This implies a full correlation between foreign and domestic demand for safe assets. The opposite assumption – making foreign demand independent of domestic conditions but the same on average – does not change the outcome noticeably.
economy, GDP increases. Strikingly however, even when foreign demand doubles the size of domestic investment in safe securities ($\gamma = 1$), which amounts to a 50% increase in total gross investment in stochastic steady-state, output only increases by 5%. By way of comparison, if the 50% increase in gross investment resulted in a 50% increase in capital formation, our aggregation result implies that GDP would increase by roughly $1.5^4 \approx 18\%$. Much like in our previous experiments, the increase in gross investment has a muted effect on output because it causes increases in security creation expenditures and producer rents, as shown in the bottom panels of Figure 10. Another similarity with our previous experiments is that TFP falls as foreign appetite for safe domestic assets increases. An increase in capital formation caused by exogenous increases in investable resources results in the activation of marginal producers which depresses average productivity.

One difference between this experiment and the experiment in which we vary the share of risk-averse investors is the key role played by producer rents. To see this, consider Figure 11 where we run our saving glut experiment with lower and higher security creation costs $\zeta$. Varying $\zeta$ has a big impact on the share of creation costs in GDP for obvious reasons, but it has little to no effect on the GDP or TFP consequences of the saving glut, because producer rents are largely unaffected. When demand for safe assets increases for exogenous reasons, marginal producers enter which increases both the mass of producers and the overall dispersion in producer talent. Aggregate rents, for both reasons, must increase.

This feature provides a potential explanation for the vast increase in financial sector rents over the past few decades documented, for instance, by Philippon and Reshef (2012). In our model, producers keep and consume their rents but one could trivially introduce intermediaries that purchase projects, pay producers the value of their outside options, pool and tranche projects as needed, and capture the resulting rents. In the such an economy, an increase in foreign demand for safe assets would result in a financial engineering boom, a significant decline in safe yields, and an increase in the rents earned by agents engaged in cash-flow transformations, all predictions borne out by the available evidence. Our model further predicts that this boom should have a muted effect on GDP and negative effect on TFP
relative to previous trends.

Another significant difference between this experiment and our other two experiments is that both security returns fall as foreign demand for safe assets rises. When only the share of domestic risk-averse investors increases, there is an excess demand for the funds risk-neutral agents supply. In contrast, an increase in foreign demand for safe assets does not have an effect on the supply of funds provided by risk-neutral agents. As capital formation rises and the marginal product of physical capital falls, the return risk-averse agents earn falls. We view this prediction as further support for the saving glut view of the recent increase since real yields have fallen across most asset classes. Importantly, the experiment does generate an increase in the premium a risk-neutral investor earn over safe assets. While the funds they provide do not become scarce in absolute terms, they do become scarce in relative terms.

Yet another interesting, if intuitively clear, consequence of the global saving glut as we model it is a divergence between national income (GNP, measured as GDP minus interest payments to foreigners) and GDP, as shown in the first panel of Figure 10. This occurs because while GDP increases little, net payments to foreigners increase markedly as foreign investment rises.

4.5 Welfare

So far we have focused entirely on the positive consequences of financial engineering booms on output, capital formation and productivity. This section discusses the consequences of these booms for the welfare of households and producers. As in Allen and Gale (1988) (or in Gennaioli, Shleifer, and Vishny (2013), when investors have rational expectations) our equilibria are constrained-efficient. A social planner that faces the same security creation costs as producers do would make the same security decisions and could not improve total surplus.

However, different agent types are affected differently by changes in security creation costs, as Figure 12 illustrates. Panel A shows household welfare (measured in expected lifetime
utility) for the two types of households and all households combined, while panel B shows the same for producers. First, total surplus (the sum of all those measures) is highest when there are not security creation costs, which should be intuitively clear. As security creation costs rise, risk-neutral agents benefit since they earn a higher premium on the portfolio of risky security they hold while, symmetrically, risk-averse agents become worse off. As we have discussed at length, selling securities to risk-neutral agents requires either bearing the tranching cost, or producers have to consume any excess payoff at a steep discount. It is no surprise, therefore, that these households are hurt when tranching costs go up, while other households benefit.

Producer welfare is measured by aggregate rents. Those rents are highest when security creation costs are zero and decline when those costs begin to increase. As panel A in Figure 3 shows, this decline coincides with a decline in the mass of active producers. Eventually however, as security creation costs continue to rise, security market clearing requires a larger share of active producers, which can only happen if rents per producer rise. Aggregate rents thus start rising once again, making the relationship between security creation costs and producer welfare non-monotonic.

Figure 13 shows the welfare consequences of increasing the fraction $\theta$ of risk-averse investors, holding the security creation cost at $\zeta = 0.1$. Once again the two household types are impacted in opposite ways, but in this experiment it is risk-neutral investors that benefit, since the relative demand for safe securities rises while fewer investors are willing to purchase risky securities. Producer welfare follows a similar pattern as in the cost experiment for similar reasons, but it is decreasing over much of the $\theta$ range.

Finally, figure 14 shows that all domestic investors are hurt by increased foreign demand for foreign securities. While wages increase slightly due to increase capital formation, the dominant effect is the decrease in rates of returns. Producers, on the other hand, unambiguously benefit from the exogenous increase in investable funds.
5 Conclusion

By allowing producers or intermediaries to create securities that appeal to investors with heterogeneous tastes, financial engineering leads to more investment broadly defined, which accords well with intuition. Less intuitive is the fact that securitization booms may not lead to much increase in output, capital formation and TFP, if any, because much of the spending on engineered securities may be dissipated into security creation costs and producer rents.

The macroeconomic impact of even large security creation booms is small in large part because costly cash-flow transformation is performed mostly by producers who are active anyway so that, in partial equilibrium, changes in the costs or benefits of security creation cannot have much of an impact on overall output and productivity. This stands in sharp contract to the traditional exercise performed in the development literature, where institutional improvements relax borrowing constraints on all agents, which has a direct effect on macroeconomic aggregates (see, for instance, Amaral and Quintin (2010), Buera and Shin (2013), Midrigan and Xu (2014) and Moll (2014).) Financial engineering booms only have an impact on the macroeconomy because of general equilibrium effects. This makes it difficult to generate big output effects, a prediction we expect to hold regardless of the details of how one chooses to model security creation.

The small effects we find, even when securitization booms come with large foreign inflows of investment funds, are also due to the fact that a significant share of those inflows goes to security creation costs and rents earned by agents who engage in cash-flow transformation. Any model of costly security creation will contain this wedge, and our results show that it can be quantitatively determinant. Our quantitative findings, in this respect, also constitute a potential explanation for the vast increase in rents associated with financial activities over the past three decades, as documented for instance by Philippon and Reshef (2012).

Finally, our results provide support for the view that the recent boom in financial engineering in the United States has been largely caused by an increase in foreign appetite for US safe assets. A drop in security creation costs can generate a large security creation
boom but, counterfactually, causes the risk-free rate to go up. An increase in the share of domestic investors who are highly risk-averse results in a boom and a fall in the risk-free rate but returns on risky investments go up, as the funds provided by less risk-averse agents become scarce, causing a large, counterfactual increase in credit spreads. Only a saving glut from abroad, where foreign demand for risk-free assets goes up, generates a boom in security creation activities without causing a large increase in credit spreads.
References


A  Proof of proposition 3

Proof. Let \{a^A, a^N, \eta_{-1}\} be the starting state of the economy at a particular date. Start with a guess \((R_N, R_A, w(B), w(G))\) for the four equilibrium prices we need, and compute the corresponding set \(Z(R_N, R_A, w(G), w(B)) \subset \mathbb{R}_+^2\) of active producers. Next, compute excess demand for each security type and labor for each of the two possible realizations of the aggregate shock in the current period. Specifically, starting with risky securities created for risk-neutral agents:

\[
ED_N(R_N, R_A, w(G), w(B)) = a^N - \int_{Z(R_N, R_A, w(G), w(B))} \frac{E(x_N(z))}{R_N} d\mu,
\]

where \(E(x_N(z))\) is the expected payoff of risky securities created by producers of type \(z \in Z(R_N, R_A, w(G), w(B))\). As for risk-averse agents:

\[
ED_A(R_N, R_A, w(G), w(B)) = a^A - \int_{Z(R_N, R_A, w(G), w(B))} \frac{x_A(z)}{R_A} d\mu,
\]

where

\[x_A(z) \in \{0, \Pi(z)\}\]

is the risk-free payoff selected by producers of type \(z \in Z(R_N, R_A, w(G), w(B))\). Excess demand for labor when the aggregate state is good is

\[
ED^L_G = \int_{Z(R_N, R_A, w(G), w(B))} n^*(w_G, z_G) d\mu - 1
\]

and the corresponding expression defines \(ED^L_B\) for the case where the aggregate shock is bad. We need to prove that \((ED_N, ED_A, ED^L_B, ED^L_G)\) is zero for at least one four-tuple of prices.

Holding other prices fixed, each element of the \(ED\) demand vector is continuous and strictly monotonic in its own price. It also diverges without bound as each price goes to zero. Existence of the zero we need follows from classical arguments. To see this, for all \(n \in \mathbb{N}\), define \(A_n = \left[ \frac{1}{n}, n \right]^4\). Then define \(G_n : ED(A_n) \mapsto A_n\) by

\[
G_n(y_1, y_2, y_3, y_4) = \arg \max_{R_N, R_A, w(B), w(G) \in A_n} w(B)y_3 + w(G)y_4 - R_Ny_2 - R_Ay_1,
\]

Roughly speaking, \(G\) raises wages when there is an excess demand for labor and lowers rates of returns when there is an excess demand for securities. The Theorem of the Maximum implies that \(G_n\) is non-empty, upper hemi-continuous and convex-valued. It follows that \(G_n \times ED\) has a fixed point on \(ED(A_n) \times A_n\).

Letting \(n\) go to \(+\infty\) gives a sequence of prices. That sequence must have a bounded subsequence. To see why, assume for instance that \(R_A\) diverges to \(+\infty\). Then at least one
wage must fall to zero. Say that \(w(B)\) goes to zero. So, then, must \(w(G)\) since otherwise there would eventually be an excess supply of labor in the good state, which, given our mapping, would mean that \(w(G)\) follows \(\frac{1}{n}\) at least along a subsequence. According to the same mapping, collapsing wages require that excess supply for labor remain positive in both states, which means that aggregate labor demand is bounded above, which means that profits are bounded above (since they are linear in the wage bill.) But then, a diverging \(R_A\) would imply that excess demand for safe securities is eventually positive, which would imply that \(R_A\) eventually follows \(\frac{1}{n}\) at least along a subsequence. Arguments are similar for the other three prices.

Since the sequence of fixed points above is bounded above and below, it must have a convergent subsequence. None of the associated prices can converge to zero. Assume for instance that \(w(B)\) did converge to zero. Given the mapping we have defined, this requires that aggregate labor demand remains below 1 in the bad state so operating profits in the bad state converge to zero. Since \(w(G)\) cannot diverge to \(+\infty\), as established in the previous paragraph, excess labor demand in the good state has to be non-negative infinitely often, which requires that \(w(G)\) also converges to zero at least along a subsequence. But then, either return would have to diverge to infinity, since otherwise, with vanishing wages, there would have to be an excess demand for labor eventually, which is incompatible with declining wages given our mapping. Again, the other prices can be dealt with using similar arguments.

It follows that along the convergent sequence of fixed points introduced above, the price part of the fixed point is eventually in the interior of \(A_n\). But given the definition of \(G_n\), this is only possible if all excess demands are zero. This completes the proof of existence. \(\square\)

\(20\) Otherwise, profits are bounded above unless demand for labor diverges to infinity (profits are linear in the wage bill) along a subsequence. If labor demand diverges, wages must converge to zero in at least one state.
Figure 1: US Asset-Backed Securities Outstanding (USD Billions)

Source: Securities Industry and Financial Markets Association (SIFMA). These volume numbers include all collateral sources except housing-related collateral.
Figure 2: **Producer policies: changing securitization costs**

**A: $\zeta=0.005$**

**B: $\zeta=0.1$**
- Risky only
- Both
- Safe only
- Inactive

**C: $\zeta=0.5$**

**D: $\zeta=0.7$**
Figure 3: Aggregate outcomes I: changing securitization costs

A: Share of active projects

B: Shares of producer types

Risky only
Safe only
Both

C: Rates of return

$\bar{R}_N$
$\bar{R}_A$

D: Tranching costs as a share of GDP
Figure 4: Aggregate outcomes II: changing securitization costs

A: Output (Index)

B: TFP (Index)

C: Wealth or wages (Index)

D: Value of risk-free debt issued (Index)

E: Rents as a share of GDP

F: Tranching issuance as a share of GDP
Figure 5: Producer policies: changing types

A: $\theta=0.1$

B: $\theta=0.5$

C: $\theta=0.7$

D: $\theta=0.9$
Figure 6: Aggregate outcomes I: changing types

A: Share of active projects

B: Shares of producer types

Risky only

Safe only

Both

C: Rates of return

\( \bar{R}_N \)

\( \bar{R}_A \)

D: Tranching costs as a share of GDP

\( \theta \)
Figure 7: Aggregate outcomes II: changing types
Figure 8: Producer policies: global saving glut
Figure 9: Aggregate outcomes I: global saving glut

A: Share of active projects

B: Shares of producer types

C: Rates of return

D: Tranching costs as a share of GDP
Figure 10: Aggregate outcomes II: global saving glut
Figure 11: High and low security creation costs: global saving glut
Figure 12: Security creation costs and welfare

Figure 13: Changing types and welfare

Figure 14: The global saving glut and welfare
Figure 15: Sensitivity analysis

A: Output (Index): changing variance

B: Output (Index): larger disaster