Cal State Fullerton
COLLEGE OF BUSINESS AND ECONOMICS
DEPARTMENT OF ECONOMICS | https://business.fullerton.edu/academics/ECONOMICS | 657-278-2228

*Department of Economics*
*Working Paper Series*

2024/001

# Are Green, Climate-change and Corporate Bonds Substitutes or Complements? Evidence from a Fourier Specification

Adrian R. Fleissig

November 2024

800 N. State College Blvd., SGMH 3313, Fullerton, CA 92831
CSUF

**Are Green, Climate-change and Corporate Bonds Substitutes or Complements? Evidence from a Fourier Specification**

Jane M. Binner, Chair in Finance
Department of Finance
University of Birmingham, England

Adrian R. Fleissig
Department of Economics
California State University, Fullerton
Fullerton, CA 92834

James L. Swofford
Department of Economics, Finance and Real Estate
University of South Alabama
Mobile, AL 36688

November 2024

**Abstract**

We estimated elasticities for climate-change, green and corporate bonds. Consistent with demand theory, own-price elasticities are negative. These assets are generally substitutes but exhibit some complementarity between climate-change and green bonds early in the sample. Thus, companies issuing bonds may want to issue both types of bonds allowing potential bond holders to diversify their portfolios. Climate-change and corporate bond budget elasticities generally exceed unity, while green bond budget elasticities are generally inelastic. These budget elasticities indicate that corporations should expect these markets to grow with economy.

**Key Words:** Climate-change Bond; Green Bonds; Corporate Bonds; Budget and Substitution Elasticities

**JEL Classifications:** G20, C14

## Introduction

Well-functioning bond markets contribute to economic development and growth. Green bonds fund a range of positive environmental projects like recycling. Climate-change bonds fund climate programs like renewable energy plants. Corporate bonds, excluding green bonds and climate change, are used for a variety of reasons. To examine changes in the demand for bonds over time, we estimate elasticities between climate-change, green and corporate bonds using a Fourier specification.

Fleissig and Swofford (1996) examined substitution between financial assets using a Fourier model, but did not include bonds. Elasticities between financial assets and bonds was examined by Fleissig and Swofford (2020) but they did not separate out climate-change and green bonds. Analyzing the impact of green bonds on other bonds is important as Tang and Zhang (2020) find stock prices respond positively to green bond issuance and liquidity improves, Flammer (2021) finds bond purchasers respond positively to green bond announcements, and Fernandes, Silva, de Araujo and Tabak (2023) find that green and corporate bonds have nonlinear correlations.

Consistent with demand theory, all estimated own-price elasticities are negative. Climate-change and green bonds are considerably more variable in the early part of the sample. The own-price elasticities of green bonds settle around those of corporate bonds, while climate-change bonds remain more elastic.

Our estimated Morishima elasticities indicate that climate-change bonds, green bonds, and corporate bonds are generally substitutes. There is evidence of complementarity between climate-change and green bonds early in the sample which implies that corporations entering these markets may want to issue both types of bonds. We reject symmetry for most of the Morishima elasticities so which bond price changes matters.

Estimated budget elasticities are positive. Climate-change bonds and corporate bonds have budget elasticities generally exceeding unity, while green bonds have inelastic budget elasticities generally between zero and unity. The budget elasticities indicate that corporations should expect these markets to grow with the economy.

## Fourier Specification

We estimated a Fourier specification to find elasticities from an unknown data generating function. The semi-nonparametric Fourier specification is dense in a Sobolev norm and can globally approximate levels and partial derivatives of a continuous utility function giving arbitrary unconstrained estimates of elasticities of substitution, see Gallant (1981) and El Badawi, Gallant, and Souza (1983). Approximating derivatives precisely is important as elasticities are derived from first and second partial derivatives.

Let $\boldsymbol{a}_t$ be a vector of real per capita quantities and $\boldsymbol{uc}_t$ a vector of nominal user-costs in period $t$. Expenditure shares for the bonds are $w_{it} = uc_{it}a_{it}/y_t$ with $\boldsymbol{v}_t = \boldsymbol{uc}_t/y_t$ the vector of expenditure normalized user-costs and $y_t = \boldsymbol{uc}_t{}'\boldsymbol{a}_t$ is total expenditure. The Fourier specification of Gallant (1981) is:

$$f(\boldsymbol{v},\boldsymbol{\theta}) = u_0 + \boldsymbol{b}'\boldsymbol{v} + \tfrac{1}{2}\boldsymbol{v}'\boldsymbol{C}\boldsymbol{v} + \sum_{\alpha=1}^{A}\left(u_{0\alpha} + 2\sum_{j=1}^{J}\left[u_{j\alpha}\cos(j\boldsymbol{k}'_{\alpha}\boldsymbol{v}) - w_{j\alpha}\sin(j\boldsymbol{k}'_{\alpha}\boldsymbol{v})\right]\right)$$

where $C = -\sum_{\alpha=1}^{A} u_{0\alpha}\boldsymbol{k}_{\alpha}\boldsymbol{k}'_{\alpha}$ and the vector of parameters to be estimated $\theta = \{\boldsymbol{b}, u_{0\alpha}, u_{j\alpha}, w_{j\alpha}: j = 1,2,\dots,J;\ \alpha = 1,2,\dots,A\}$. A multi-index, $\boldsymbol{k}_{\alpha}$, denotes partial differentiation of the utility function. The number of terms and degree of the Fourier polynomials are determined by the parameters $A$ and $J$ through empirical testing. The nonlinearity of the data generating function will determine the number of terms used and degree of the polynomials. The share equations are:

$$w_i(\boldsymbol{v},\boldsymbol{\theta}) = \frac{b_i v_i - \sum_{\alpha=1}^{A}\left(u_{0\alpha}\boldsymbol{v}'\boldsymbol{k}_\alpha + 2\sum_{j=1}^{J} j\left[u_{j\alpha}\sin(j\boldsymbol{k}'_\alpha\boldsymbol{v}) + w_{j\alpha}\cos(j\boldsymbol{k}'_\alpha\boldsymbol{v})\right]\right) k_{i\alpha} v_i}{\boldsymbol{b}'\boldsymbol{v} - \sum_{\alpha=1}^{A}\left(u_{0\alpha}\boldsymbol{v}'\boldsymbol{k}_\alpha + 2\sum_{j=1}^{J} j\left[u_{j\alpha}\sin(j\boldsymbol{k}'_\alpha\boldsymbol{v}) + w_{j\alpha}\cos(j\boldsymbol{k}'_\alpha\boldsymbol{v})\right]\right) \boldsymbol{k}'_\alpha\boldsymbol{v}}$$

With three or more variables Blackorby and Russell (1989) show that the elasticity of substitution is measured by the Morishima elasticity $ME_{ij} = w_i\,(\sigma_{ji} - \sigma_{ii})$ where $\sigma_{ij}$ are the Allen elasticity of substitution. The Morishima elasticity provides estimates of substitution for both changes in the price of asset $i$ and $j$ and has been used by Davis and Gauger (1996), Fisher and Fleissig (1997), Fleissig and Swofford (2022), Liu and Serletis (2022) and Serletis and Xu (2023).

## Data

We use monthly data on U.S. asset and returns for climate-change, green and corporate bonds. Climate-change and green bond data are from Bloomberg[1] with corporate bonds holdings from FRED.[2] The sample has 46 monthly observations from 2019:1 through 2022:10 for each of the three bonds.[3] As green bonds include climate-change bonds, we subtracted climate-change bonds from green bonds to find non-climate-change green bonds. Similarly, since climate-change and green bonds are corporate bonds, we subtracted green bonds from corporate bonds to find non-green corporate bonds. We converted the bonds data into real per capita terms using the monthly CPI-U and monthly population 16-years old and older.

Following Barnett (1978) and Donovan (1978) the nominal user-costs for each asset are:

$$uc_{it} = P_t(R_t - r_{it})/(1 + R_t),$$

where $r_{it}$ is the return on an asset, $P_t$ is the CPI and $R_t$ is a benchmark rate. Rates of returns on climate-change and green bonds are the minimum of yield to call and yield to maturity of the

[1] H33588US Index (Bloomberg climate change bonds proxy index US) and I31563US Index (Bloomberg US green bond index_corporate).
[2] CFBABSHNO is quarterly and monthly observations were interpolated.
[3] Or 138 observations over the three-equation specification.

associated bonds from Bloomberg.[4] The rate of return on corporate bonds is the monthly 10-year high quality market corporate bond par yield from FRED.[5] The benchmark rate is calculated as the maximum return from all the assets following Anderson and Jones (2011).

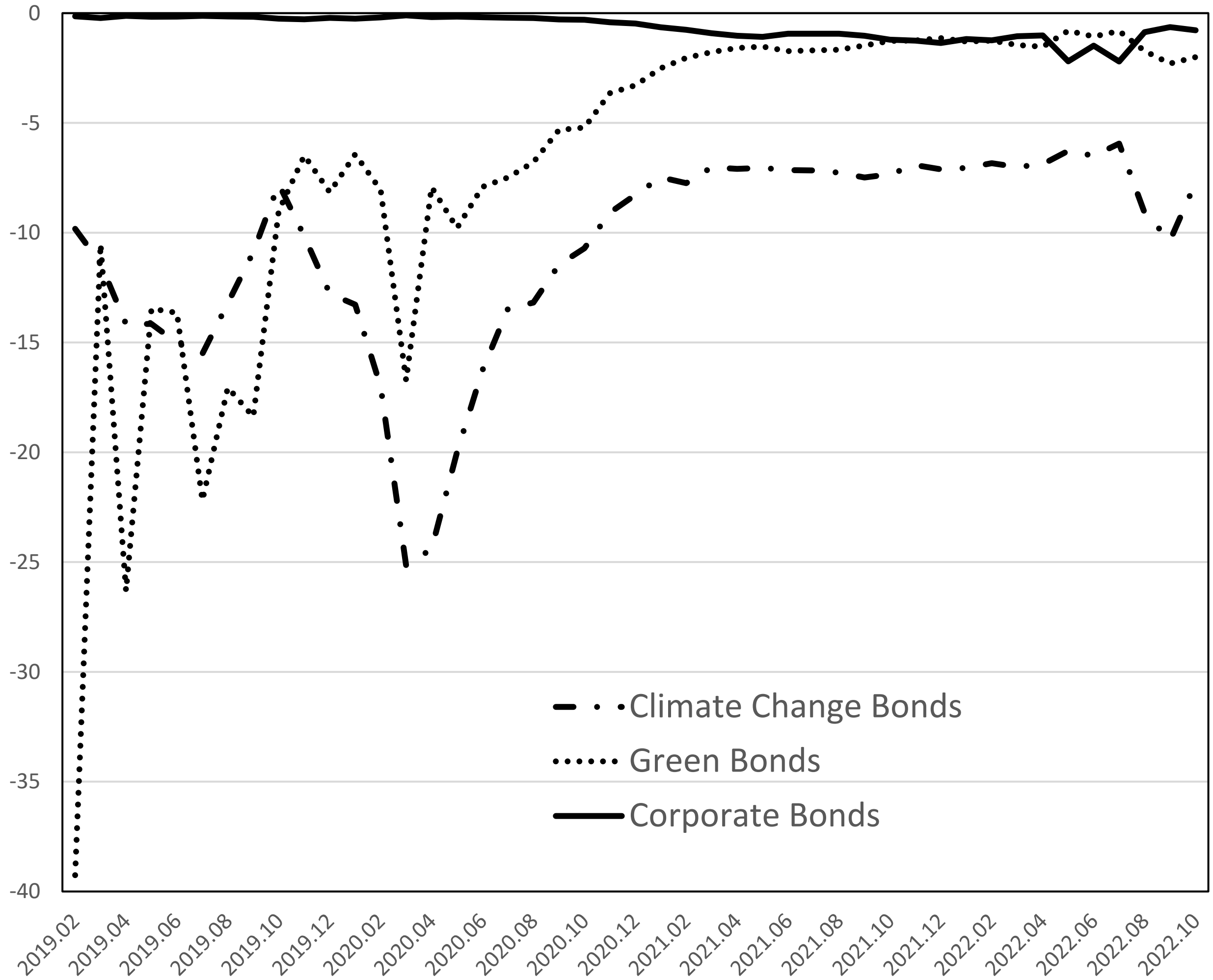
## Results

We estimated a Fourier specification using International TSP 5.1 seemingly unrelated regression procedure with across-equation restrictions imposed to ensure adding up, as in Gallant (1981).[6] The normalization of $b_3$=-1 was imposed with convergence set at 0.00001. The degree of the Fourier polynomials, A=3 and J=1, was determined by the upward F-test procedure of Eastwood and Gallant (1991). Most parameters are statistically significant and share equations provide an accurate approximation in terms of the R-square and the root mean square error. A first-order vector autoregressive process is applied to correct for serial correlation (Berndt and Savin, 1975). A Q-statistic with the Box-Pierce test for autocorrelation for each share indicates white noise at the 5% level.

Consistent with demand theory, the own-price (user-cost) elasticities for each asset are always negative.[7] In Figure 1 corporate bonds are generally inelastic in demand, although for some months slightly elastic. Green bonds are generally elastic and perhaps unsurprisingly for a new market considerably more variable in the early months of the sample before settling into a similar range to corporate bonds. Climate-change bonds are elastic and also considerably more variable early in the sample. Climate-change bonds own-price elasticities remain more elastic, but in the range found by Fleissig and Swofford (2020) for financial assets.

[4] Minimum of yield to call and yield to maturity is sometimes called yield to worst.
[5] Series HQMCB10YRP.
[6] All results discussed but not presented are available upon request.
[7] All 138-point estimates of the own-price elasticities are negative.

**Figure 1**
**Own Elasticities**




Descriptive statistics for the Morishima elasticities are in Table 1. Climate-change, green and corporate bonds are substitutes on average, but for some months early in the sample climate-change and green bonds were complements. Corporations entering these markets may want to issue both types of bonds allowing bond holders to diversify their portfolios.

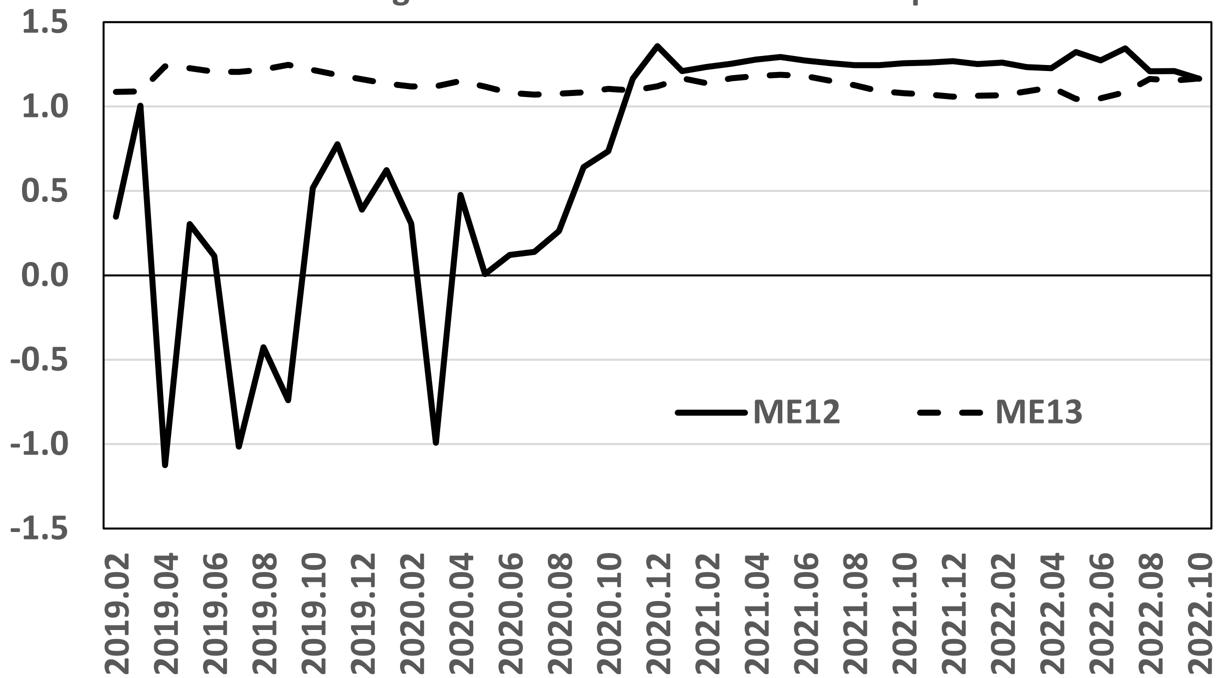
**Table 1**
**Morishima Elasticities of Substitution**

| Elasticity | Mean | Minimum | Maximum | Standard Deviation |
|---|---|---|---|---|
| ME12 | 0.723 | -1.124 | 1.358 | 0.711 |
| ME21 | 0.864 | -1.358 | 2.155 | 0.966 |
| ME13 | 1.132 | 1.045 | 1.247 | 0.056 |
| ME31 | 1.662 | 0.757 | 2.516 | 0.442 |
| ME23 | 1.179 | 0.445 | 1.622 | 0.256 |
| ME32 | 1.836 | 1.125 | 2.536 | 0.351 |

$ME_{ij}$=Morishima Elasticity between asset $i$ and $j$ for a change in the user-cost of asset i
1=Climate-change Bonds 2=Green Bonds 3=Corporate Bonds

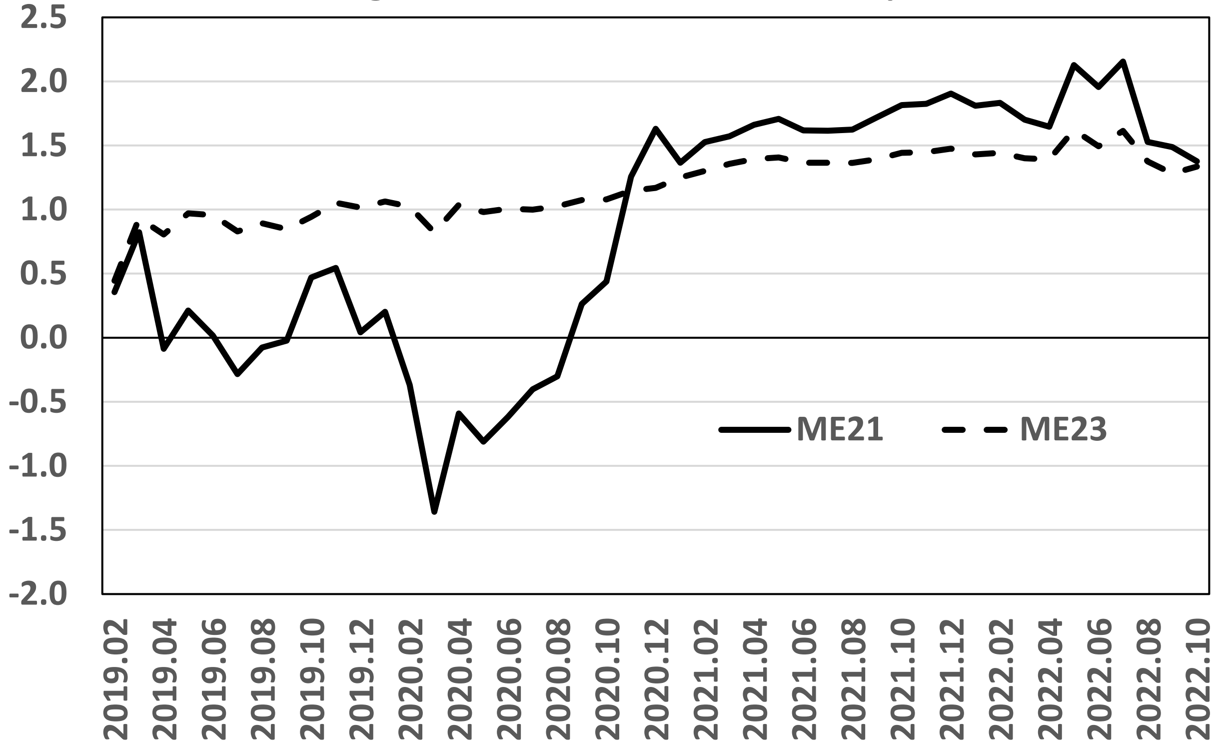
Morishima elasticities between climate-change and green bonds (ME12), for a change in the user-cost of climate-change bonds are in Figure 2. The ME12 elasticities are relatively variable showing considerable complementarity in early periods when markets are still new and relatively thin. These assets become substitutes as these markets mature and deepen. Elasticities between climate-change bonds and corporate bonds (ME13) show substitution over the entire sample.

Figure 2
ME12 and ME13
1=Climate-change Bonds   2=Green Bonds   3=Corporate Bonds



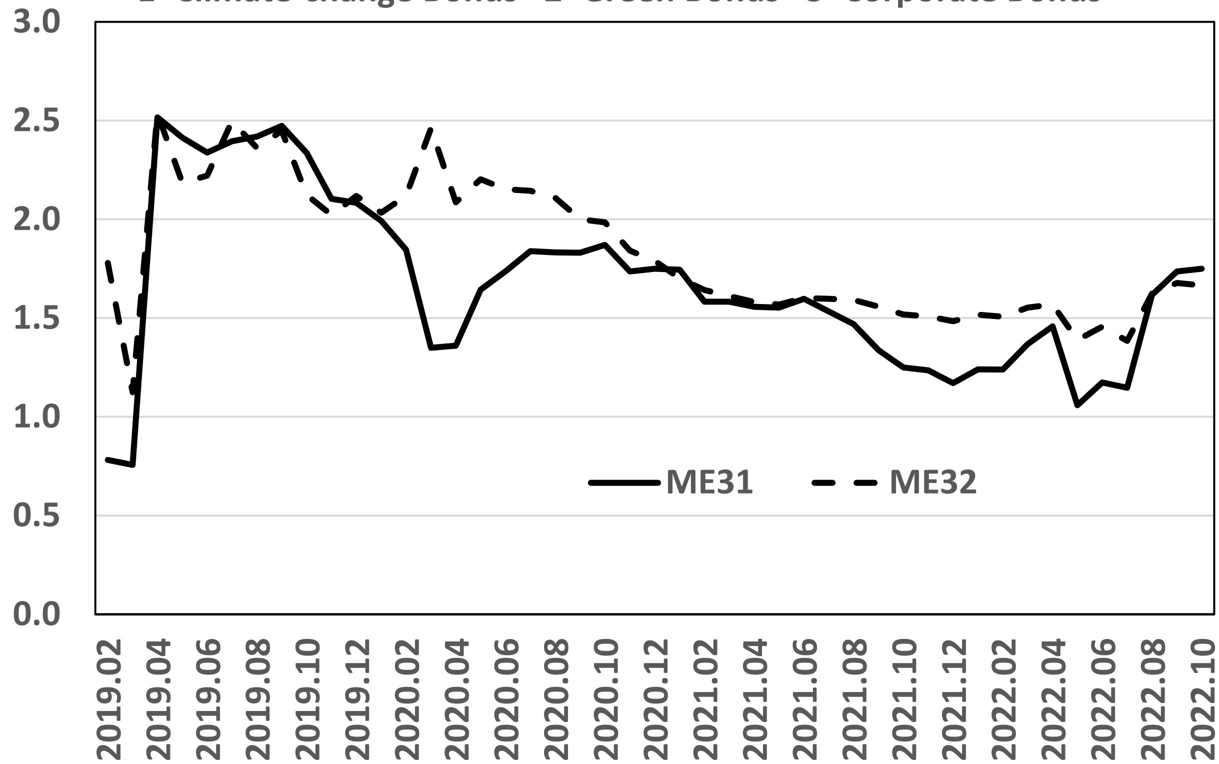

Again, Morishima elasticities between green bonds and climate-change bonds (ME21) shown in Figure 3 are considerably more variable in the early part of the sample and show some periods of complementarity.  Morishima elasticities between green and corporate bonds (ME23), for a change in the user-costs of green bonds, are less variable and always substitutes.

**Figure 3**
**ME21 and ME23**
**1=Climate-change Bonds   2=Green Bonds   3=Corporate Bonds**



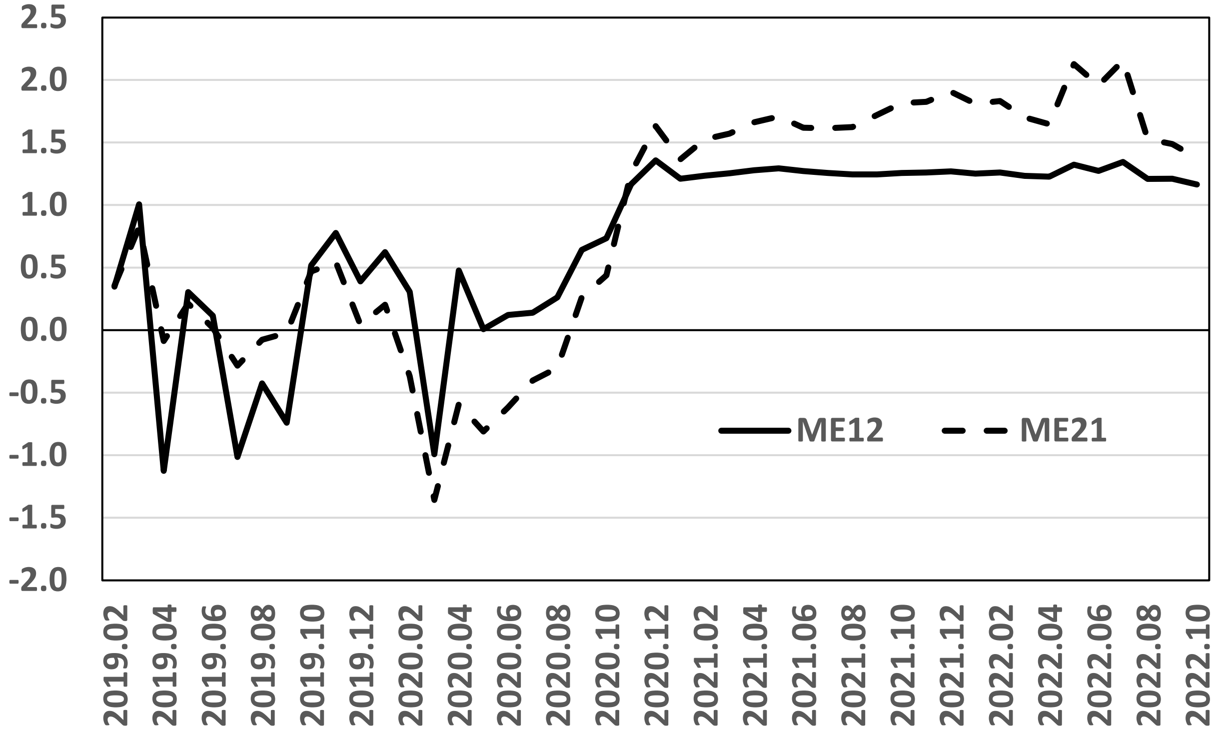

The Morishima elasticities between corporate and climate-change bonds (ME31) and corporate and green bonds (ME32), for a change in the user cost of corporate bonds are in Figure 4. These assets are substitutes in use for each other in each period and is relatively more variable in 2019 compared to the remainder of the sample.

**Figure 4**
**ME31 and ME32**
**1=Climate-change Bonds   2=Green Bonds   3=Corporate Bonds**



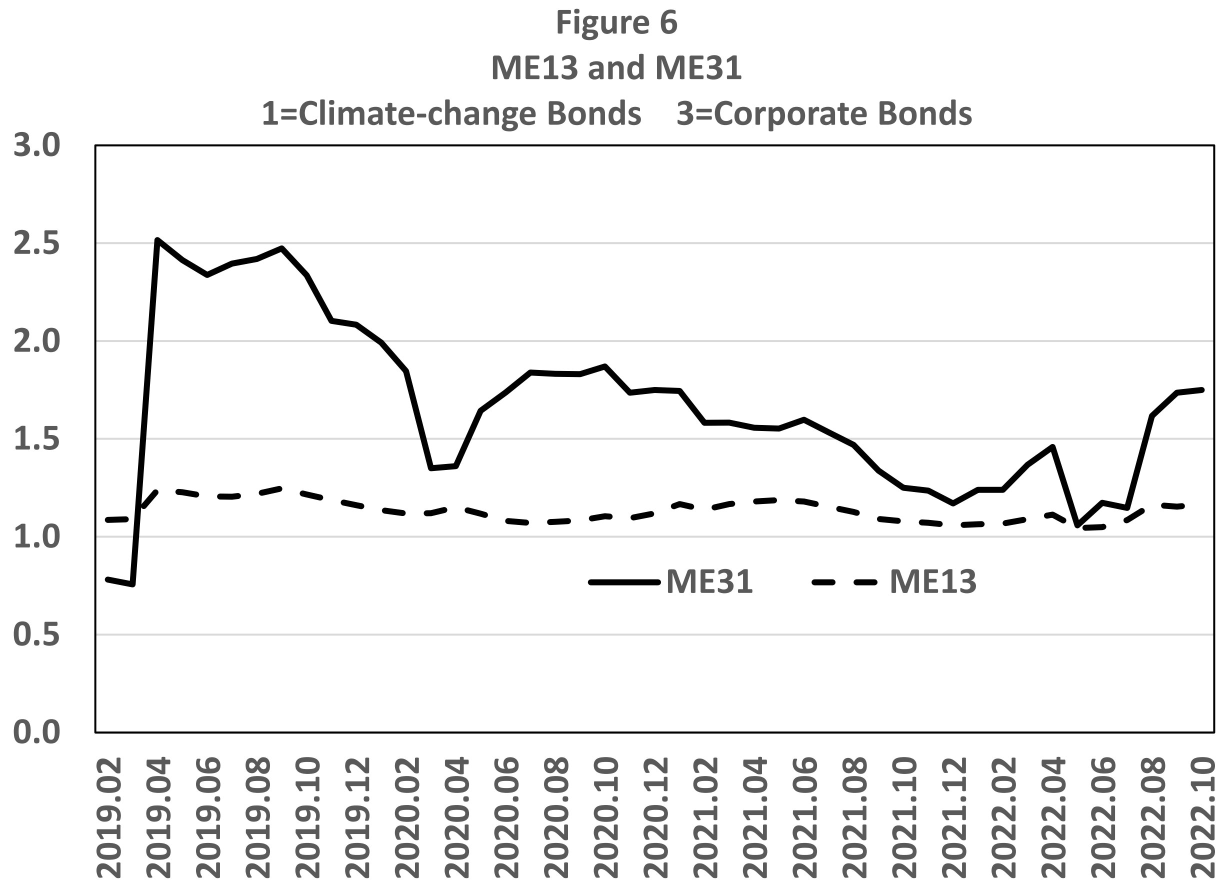

Morishima elasticities are not required to be symmetric and can differ depending on if asset *i* or *j* causes a change in the relative user-cost. We test for symmetry by evaluating if MEij=MEji and reject symmetry for 107 out of 132 pairs of Morishima elasticities. Thus, changes in the user cost of each bond can have different impacts on substitution between pairs of bonds.

**Figure 5**
**ME12 and ME21**
**1=Climate-change Bonds 2=Green Bonds**



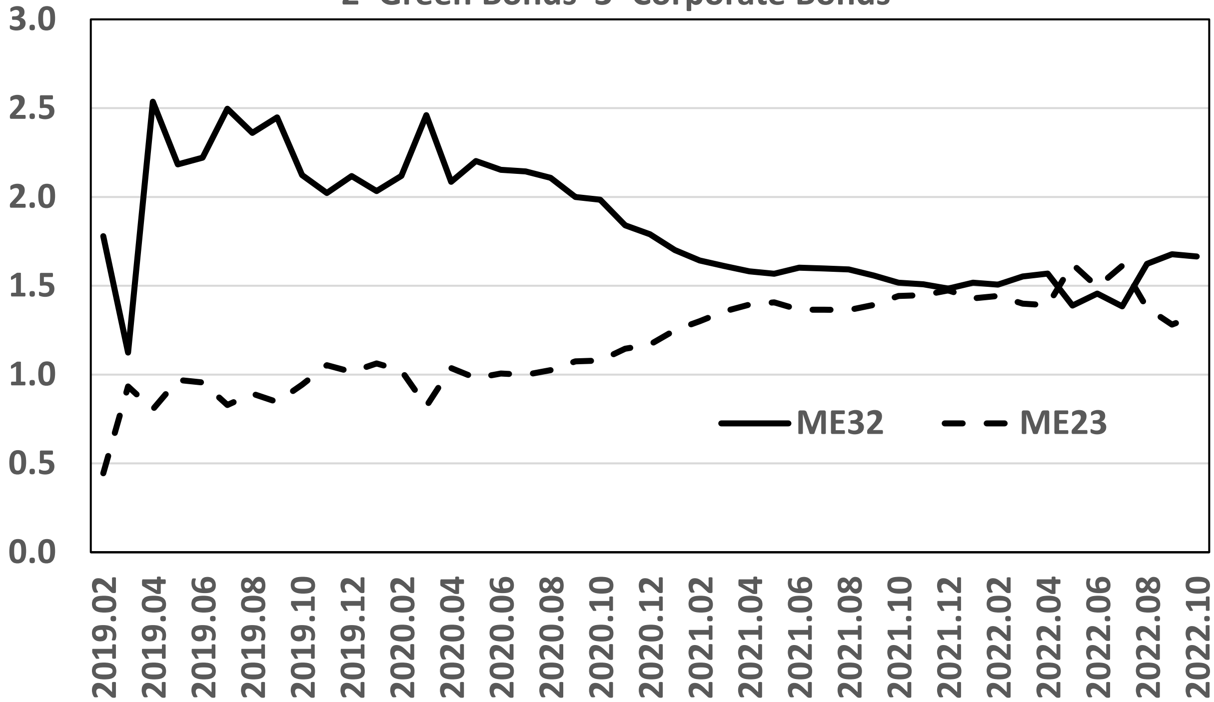

The Morishima elasticities between climate-change and green bond are in Figure 5 and show complementarity for 11 months for 2019 and 2020. The Morishima elasticity of climate-change bonds with respect to changes in the user cost of green bonds (ME21) shows complementarity for five early months. We reject symmetry for 37 of 44 for the ME12 and ME21 elasticities.

Climate-change bonds and corporate bonds are always substitutes as shown in Figure 6. The Morishima elasticities of corporate bonds, with respect to change in the user-cost of climate-change bonds (ME13), are relatively less variable and always exceed unity. The elasticities of climate-change bonds, with respect to corporate bonds (ME31), while always substitutes vary considerably more and fall below unity in some early periods. We reject symmetry for 37 of 44 Morishima elasticities.



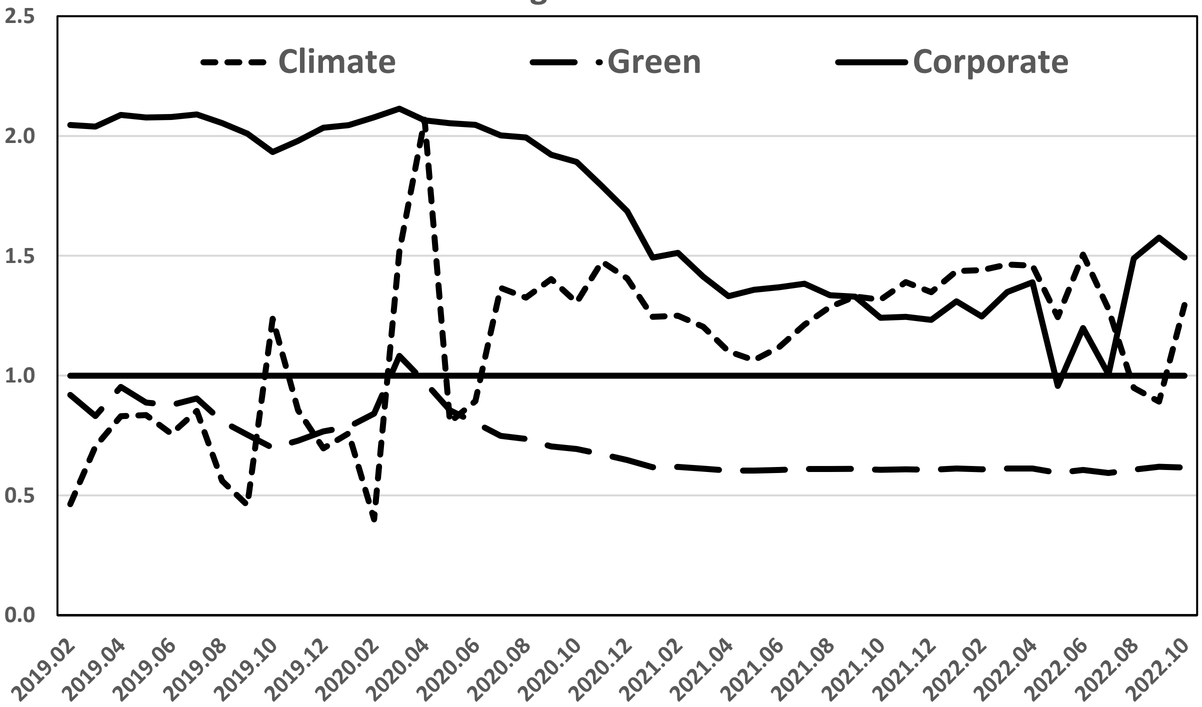

Morishima elasticities between green and corporate bonds show that they are always substitutes as shown in Figure 7. The degrees of substitution are quite different early on in the sample but appear to converge as the green bond market matures and are quite similar at the end of the period. We reject symmetry for 33 of 44 of these elasticities.

Figure 7
ME23 and ME32
2=Green Bonds 3=Corporate Bonds




The budget elasticities shown in Figure 8 are positive for each asset. Climate-change and corporate bonds have budget elasticities generally exceeding unity. Green bonds have budget elasticities that are generally inelastic and between zero and unity. Thus, as the economy grows, climate-change and corporate bonds purchases grow by a larger percentage than green bonds. These budget elasticities indicate corporations should expect these markets to grow with the economy.

**Figure 8**
**Budget Elasticities**




## Conclusions

We estimate elasticities between climate-change, green and corporate bonds. Consistent with demand theory, the estimated own-price elasticities were all negative. Own-price elasticities of climate-change and green bonds are considerably more variable early in the sample when those markets were relatively new and thin. Own-price elasticities of corporate bonds are inelastic on average but close to and sometimes exceed unity.

Morishima elasticities on average show that bonds are substitutes for each other. There is complementarity between climate-change bonds and green bonds early in the sample. Thus, corporations may want to issue green and climate-change bonds when they enter these markets.

The budget elasticities are all positive indicating corporations should expect these markets to grow as the economy grows. Climate-change and corporate bond budget elasticities generally exceed unity, while green bond budget elasticities were inelastic and between zero and unity.

**References**


Anderson, R.G. and Jones, B.E., 2011. A Comprehensive Revision of the U.S. Monetary Services (Divisia) Indexes. Federal Reserve Bank of St. Louis Rev. 93(5), 325-359.

Barnett, W.A., 1978. The User-cost of Money. Economics Letters, 1(2), 145-149.

Berndt, E.R. and Savin, N.E., 1975. Estimation and Hypothesis Testing in Singular Equation Systems with Autoregressive Disturbances. Econometrica 43, 937–957

Blackorby, C. and Russell, R., 1989. Will the Real Elasticity of Substitution Please Stand Up? (A Comparison of Allen/Uzawa and Morishima elasticities). American Economic Review, 79 (4), 882–888.

Davis, G.C. and Gauger, J., 1996. Measuring Substitution in Monetary-Asset Demand Systems Journal of Business and Economic Statistics, 14, 203-208

Donovan, D. J., 1978. Modeling the Demand for Liquid Assets: An Application to Canada. International Monetary Fund IMF Staff Papers 25(4), 676-704.

Eastwood, B. J. and Gallant, A.R., 1991. Adaptive Rules for Semiparametric Estimators that Achieve Asymptotic Normality. Econometric Theory. 7(3), 307-340.

El Badawi, I., Gallant, A.R. and Souza, G., 1983. An Elasticity Can be Estimated Consistently without a priori Knowledge of Functional Form. Econometrica 51(6), 1731-1751.

Fatica, S., Panzica, R., 2021. Green bonds as a tool against climate-change? Business Strategy and the Environment. 30 (5), 2688–2701.

Fisher, D., & Fleissig, A. R. (1997). Monetary aggregation and the demand for assets. Journal of Money, Credit, and Banking, 458-475.

Fernandes, L.H.S. and J. W.L. Silva, F.H.A. de Araujo and B.M. Tabak (2023). Multifractal cross-correlations between green bonds and financial assets, Finance Research Letters, 103603.

Flammer, C., 2021. Corporate green bonds, Journal of Financial Economics, 142(2), 499-516.

Fleissig, A.R. and Swofford, J. L. (1996). A Dynamic Asymptotically Ideal Model of Money Demand. Journal of Monetary Economics, 37, 371-380.

Fleissig A.R. and Swofford J. L. 2020 The demand for assets through a low-interest rate environment, Applied Economics, 52:60, 6540-51

Gallant, A.R., 1981. On the Bias in Flexible Functional Forms and an Essentially Unbiased Form: the Fourier Flexible Form. Journal of Econometrics 15 (2), 211–245.

Liu, J., and Serletis, A. (2022). Credit cards, the demand for money, and monetary aggregation. Macroeconomic Dynamics, 26(8), 2161-2203.

Serletis, A., and Xu, L. (2023). Consumer preferences, the demand for Divisia money, and the welfare costs of inflation. Journal of Macroeconomics, 75.

Tang, D. Y. and Zhang, Y., (2020). Do shareholders benefit from green bonds? Journal of Corporate Finance, 101427.