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Age-Dependent Increasing Risk Aversion and the Equity Premium Puzzle[1]

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Abstract

We introduce a new preference structure—age-dependent increasing risk aversion (IRA)—in a three-period overlapping generations model with borrowing constraints, and examine the behavior of equity premium in this framework. We find that IRA preferences generate results that are more consistent with U.S. data for the equity premium, level of savings and portfolio shares, without assuming unreasonable levels of risk aversion. We find that the relative difference between the two risk aversions (how much more risk-averse old agents are relative to the middle-aged) matters more than the average risk aversion in the economy (how much more risk averse both cohorts are). Our findings are robust with respect to a number of model generalizations.

JEL Classification: G0, G12, D10, E21.

Key Words: Equity premium puzzle, Overlapping generations model, Increasing Risk Aversion, Portfolio allocation.

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1. Introduction

The equity premium puzzle, first presented in the seminal work of Mehra and Prescott (1985), underscores the inability of standard, reasonably parametrized representative-consumer exchange models to match the historical equity premium observed both in the United States and in international markets. The essence of the puzzle is driven by the definition of risk: the model parsimoniously links the risk premia of financial assets with per capita consumption growth. As it is now well understood, this covariance is typically one order of magnitude lower than what is needed to generate the observed equity premium, implying that the price of risk must be implausibly high to reconcile model predictions with its historical counterpart. Subsequent studies have shown that the puzzle is robust (Campbell, 1996; Kocherlakota, 1996), and it is neither a country-specific phenomenon (Campbell, 2003; Mehra and Prescott, 2003) nor a period-specific one (Siegel, 1992).

Aspects of the equity premium puzzle are manifested in the failure of standard models to replicate other key empirical regularities. For example, the observed stock price volatility is too high to be matched by the smoothed dividend process observed in the data (Shiller, 1981). Moreover, standard models, calibrated at historical (high) levels of equity premium and moderate (known) levels of risk aversion, produce a counterfactually high demand for equity forcing theoretically optimal portfolios to be much more heavily invested in stocks than what the data suggest. In most cases, these models predict that the appropriate proportion of wealth invested in the risky asset is close to 100% (Benzoni, Collin-Dufrense, and Goldstein, 2007), while the average share of stocks in financial portfolios in the data is only slightly above 50% (Bertaut and Starr-McCluer, 2002).

Not surprisingly, the equity premium puzzle has spawned a voluminous body of research aimed at reconciling the high equity premium observed in the data with the theoretical findings of plausibly specified asset pricing models. Several generalizations of the key features of the Mehra and Prescott (1985) model have been proposed, ranging from preference modifications, lower tail risks, survival bias, incomplete markets, market imperfections, limited participation, macroeconomic shocks, long-run growth prospects, and behavioral explanations.[1] Many of these generalizations have contributed importantly to our understanding of the puzzle and the dynamics driving asset pricing. However, even though enormous progress has been made in reconciling facts with theory, no single unified theory appears to have solved all aspects of the puzzle.

This paper explores whether the equity premium puzzle can be explained in a parsimonious asset pricing model with preference heterogeneity over the life cycle and borrowing constraints. Our basic framework is the overlapping generations (OLG) model of Constantinides, Donaldson, and Mehra (CDM) (2002) with borrowing constraints. The novelty of this study lies in introducing a new preference structure—age-dependent increasing risk aversion (IRA)—in this setting: agents become more risk averse as they age. This type of preference heterogeneity is motivated by a large number of empirical and survey-based studies that have routinely documented a strong positive relation between age and risk aversion.[2]

Following CDM (2002), we assume that there are three age cohorts (young, middle aged, and old), each facing different sources of uncertainty on wage and equity income. The attractiveness of equity depends on the stage of the life cycle. The young, for whom equity is attractive as a hedge against future consumption fluctuations, would like to borrow to invest in equity but are unable to do so due to the borrowing constraint. Equity does not have the same appeal for the middle-aged given that consumption fluctuations are entirely driven from fluctuations in equity income. Therefore, the constraint reduces the risk-free rate (because the young are unable to borrow), increases equity returns

[1] See, for example, Rietz (1988), Weil (1989), Constantinides (1990), Epstein and Zin (1990), Telmer (1993), Heaton and Lucas (1997), Basak and Cuoco (1998), Campbell and Cochrane (1999), Constantinides, Donaldson, and Mehra (2002), McGrattan and Prescott (2003), Bansal and Yaron (2004), Barro (2006), Guvenen (2009), DaSilva, Farka, and Giannikos (2011), Gârleanu and Panageas (2015), and Abbot (2017).

[2] See, for example, Morin and Suarez (1983), Riley and Chow (1992), Bakshi and Chen (1994), Lee and Hanna (1995), Palsson (1996), and Sung and Hanna (1996).

(because the middle-aged require a higher premium) resulting in a higher premium. However, even though it goes a long way, the borrowing constraint fails to fully account for key aspects of the data: the predicted equity risk premium falls short of the historical average, the level of savings is higher than observed, and portfolio shares tend to be more heavily skewed toward the risky asset than they are in practice.

These shortcomings are largely resolved once we introduce age-dependent IRA into the standard CDM framework with borrowing constraints. We find that the introduction of IRA preferences in a life-cycle model plays a key role in determining the price of risk in the economy, matching the equity premium while simultaneously delivering portfolio allocations that are more closely aligned with the data: the equity premium is in line with the historical average (6%–7%), and the portfolio share of the risky asset is in the 40% to 50% range. Importantly, these results are obtained for fairly moderate levels of risk aversion. These results are generally robust with respect to a number of model extensions (correlation structure, scale changes, growth, and pension schemes) and the main message from this work—that an OLG model with IRA preferences delivers results that better match empirical data—remains essentially unchanged under these alternative specifications.

IRA preferences effectively introduce an additional source of risk in the model in addition to the usual pure consumption uncertainty, thus sidestepping one of the key aspects of the equity premium puzzle: low consumption (growth) volatility. Small variations in consumption are now amplified by the steeper risk-aversion profile over the life cycle leading to a substantial increase in consumption volatility for the middle-aged agents (the marginal investor). Faced with an increasing risk aversion over the life cycle, the IRA agents are now even more averse to gambles that play out in the future, so they saves less (savings effect) and tilt portfolio shares away from equity and toward bonds (portfolio substitution effect).

Our results show that a small amount of preference heterogeneity over the life cycle goes a long way toward delivering model predictions that are more in line with empirical data. We also find that the difference between the two risk aversions (how much more risk averse old agents are relative to the middle-aged) matters more than the average risk aversion in the economy (how much more risk averse both cohorts are). An economy with low average risk aversion but where agents become significantly more risk averse as they age delivers a much higher premium than an economy with a higher overall risk aversion but flatter profile. This occurs because for a relatively large increase in risk aversion over the life cycle, the relative outlook variation increases even when the average risk aversion in the economy is low. In a sense, the lifetime risk profile dominates the average economy-wide risk aversion, amplifying the savings and portfolio substitution effects.

A large body of work has examined the effect of preference heterogeneity on asset prices. In particular, a number of studies assume heterogeneity in risk aversion when agents have expected utility (Dumas, 1989; Wang, 1996; Basak and Cuoco, 1998; Kogan, Markov, and Uppal, 2007; Weinbaum, 2012; Abbot, 2017). Others assume heterogeneity in risk aversion when agents exhibit habit-forming preferences (Chan and Kogan, 2002; Xiouros and Zapatero, 2010; DaSilva, Farka, and Giannikos, 2011; Bhamra and Uppal, 2014). A few other studies extend preference heterogeneity into two dimensions with recursive Epstein-Zin preferences: heterogeneity in risk aversion and elasticity of intertemporal substitution (Gomes and Michaelides, 2008; Guvenen, 2009; Gârleanu and Panageas, 2015). Last, a number of papers use representative agent models with state-dependent preferences where the coefficient of risk aversion varies with the state of the economy (see Danthine et al. [2004] for an application with coefficient of relative risk aversion [CRRA] preferences and Melino and Yang [2002] for one with recursive preferences).

This study further extends the current literature by introducing a new preference type where investors' risk aversion varies over the life cycle. To our knowledge, this paper is the first to consider this type of preference modification. Age-dependent IRA preferences add another dimension to the across-generation heterogeneity of CDM (2002), while preserving the elegance and the economic insights that follow from that model.

In addition, most papers with cross-section heterogeneity in risk aversion require a high degree of heterogeneity between agents—that is, the most risk-averse type needs to be extremely risk averse (Kogan, Markov, and Uppal, 2007; Weinbaum, 2012; Gârleanu and Panageas, 2015; Abbot, 2017). In contrast, our results obtain for small differences in risk aversion over the life cycle. Moreover, our IRA specification has other advantages relative to other, more commonly used preference modifications. For example, unlike habit formation which requires unrealistically high levels of effective risk aversion (Constantinides, 1990; Campbell and Cochrane, 1999; Chan and Kogan, 2002), our model is able to produce results more in line with empirical data without assuming unreasonable parameter values. Also, IRA preferences do not seem to suffer from the same issues as state-dependent preferences, which tend to deliver extremely high equity returns and asset price volatilities (Danthine et al., 2004). Finally, the IRA agent in our model is indifferent as to the timing of the resolution of uncertainty, an issue that is inherent in Epstein-Zin preferences.[3]

The rest of this paper is organized as follows. Section 2 outlines the model and calibration. Section 3 presents our main findings: results from our IRA model are compared to the CRRA baseline model of CDM (2002). Various extensions of the baseline model are presented and discussed in Section 4. Concluding remarks are summarized in Section 5.

[3]Epstein, Farhi, and Strzalecki (2014) show that with recursive preferences, an investor must be willing to pay a very large amount for the early resolution of uncertainty. This means using recursive preferences to solve the equity premium puzzle leads to another puzzle.

2. Model and calibration

2.1 A new preference specification: age-dependent increasing risk aversion

We consider the borrowing-constrained version of the three-period OLG exchange economy of Constantinides, Donaldson, and Mehra (2002), where each generation is modeled by a representative agent and lives as young, middle-aged, and old. There is only one consumption good (perishable at the end of each period); wages, consumption, dividends, and coupons payments are quoted in terms of this single consumption good.

Two types of securities are traded: a bond and a share of equity. The bond is a default-free asset that pays a fixed coupon of one unit of the consumption good in every period in perpetuity. The aggregate coupon payment is b in every period (its supply is fixed at b units) and represents a portion of the economy's capital income. p_t^b is the ex-coupon price of bond in period t. The equity is the claim to the net dividend stream $\{d_t\}$: the sum total of all the private capital income (stocks, corporate bonds, and real estate). Its supply is fixed at one share in perpetuity. The ex-dividend price of equity in period t is p_t^e. x_t^b and x_t^e denote the share invested in bonds and equity, respectively.

The consumer born in period t receives a low deterministic wage income w^0 in period t (when young), stochastic wage income w_{t+1}^1 in period $t+1$ (when middle-aged), and zero wage income in period $t+2$ (when old). The young start out with zero endowment of bonds and equity. Faced with current low deterministic wages and uncertain future wages, they would like to hedge income risk by borrowing against future wages, consuming part of the loan, and investing the rest in equity. However, in the borrowing-constrained version of the CDM (2002) economy, the young are prevented from short-selling bonds because, as argued by the authors, human capital alone does not constitute adequate collateral for loans due to adverse selection and moral hazard issues.[4] Thus, in the

[4] The borrowing constraint on the young, much as in the CDM (2002) work, is exogenously imposed rather than endogenously determined. A more realistic approach would be to allow for uninsurable, persistent, and heteroskedastic labor income shocks that would deter the young consumers from investing in equity (Storesletten, Telmer, and Yaron, 2007) or for a small probability of a disastrous labor income outcome (Cocco, Gomes, and Maenhout, 2005). We abstract from modeling labor income risk (leaving

borrowing-constrained economy there exists a rational expectations equilibrium in which the young do not participate in the bond and equity markets—that is, $x^b_{t,0}$ and $x^e_{t,0}$ are zero.[5]

For the middle-aged, the perspective is different: these investors do not view equity as favorably as the young. At this stage of the life cycle, their wage uncertainty is resolved and consumption is highly correlated with equity income. However, faced with zero income in the next period (when old), the middle-aged agent optimally decides to save and invest in a diversified portfolio of stocks and bonds in order to smooth consumption over the life cycle, purchasing $x^b_{t,1}$ and $x^e_{t,1}$ shares of bonds and equity. Investment positions are liquidated and consumed when the agent is old ($x^b_{t,2} = 0$ and $x^e_{t,2} = 0$).

The dynamic budget constraint for the representative consumer in this borrowing-constrained economy varies with the stage of the life cycle. Denoting by $c_{t,j}$ the consumptions in period $t + j$ ($j = 0,\ 1,\ 2$) of a consumer born in period t, the budget constraint is given by

$$c_{t,0} \leq w^0 \qquad \text{when young} \tag{1a}$$

$$c_{t,1} \leq w^1_{t+1} - x^b_{t,1} p^b_{t+1} - x^e_{t,1} p^e_{t+1} \qquad \text{when middle-aged} \tag{1b}$$

$$c_{t,2} \leq x^b_{t,1}(p^b_{t+2} + 1) + x^e_{t,1}(p^e_{t+2} + d_{t+2}) \qquad \text{when old} \tag{1c}$$

where $c_{t,0} \geq 0$, $c_{t,1} \geq 0$, and $c_{t,2} \geq 0$, thus ruling out negative consumption and personal bankruptcy.[6]

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it for future research) in order to retain the basic features of the baseline CDM model while highlighting the role of IRA preferences.

[5] The young are allowed to borrow by shorting equity, but as in CDM (2002), the restriction on shorting equity is nonbinding for the range of parameters used in calibration.

[6] In the unconstrained version of the economy, the budget constraints are:

$$c_{t,0} + x^b_{t,0} p^b_t + x^e_{t,0} p^e_t \leq w^0$$

$$c_{t,1} + x^b_{t,1} p^b_{t+1} + x^e_{t,1} p^e_{t+1} \leq w^1_{t+1} + x^b_{t,0}(p^b_{t+1} + 1) + x^e_{t,0}(p^e_{t+1} + d_{t+1})$$

$$c_{t,2} \leq x^b_{t,1}(p^b_{t+2} + 1) + x^e_{t,1}(p^e_{t+2} + d_{t+2}).$$

These reduce to (1a)–(1c) in a borrowing-constrained economy because the young do not participate in financial markets ($x^b_{t,0} = 0$ and $x^e_{t,0} = 0$). As such, the borrowing constraint reduces the setup into a

With this setup, the three-period OLG model of CDM (2002) has three distinct age cohorts: the borrowing-constrained young, the saving middle-aged, and the dissaving old. As argued by Constantinides, Donaldson, and Mehra (2002), the borrowing constraint increases equity returns relative to an unconstrained economy because securities are priced solely by the middle-aged investors for whom equity is not as attractive. At the same time, the inability of the young to borrow by short-selling bonds lowers bond returns. In equilibrium, both effects combine for a higher risk premium than in the unconstrained economy.

Although the introduction of the borrowing constraint goes a long way toward the resolution of the puzzles, it does not fully resolve them. With CRRA preferences (as in CDM [2002]), the model delivers an equity premium in the range of 2.2% to 4.7% (for CRRA values of 2 to 6), which falls short of the historical average of around 6%. Moreover, the level of savings as a share of income is unrealistically high at roughly 30%, while the macroeconomic evidence points to a much lower range of 10% to 20%. The share of equity in the optimal portfolio is also higher than what is commonly found in empirical observations, 63% versus 45% to 55%.

The novelty of this study lies in introducing age-dependent IRA into the OLG economy of CDM (2002) with borrowing constraints. This adds another source of heterogeneity to the CDM model, in addition to the agent heterogeneity represented by idiosyncratic endowment shocks in their framework. Specifically, the consumer born in period t has utility

$$E\left(\sum_{i=0}^{2}\beta^{i}u\left(c_{t,i},\alpha_{i}\right)|I_{t}\right), \tag{2a}$$

with

$$u(c_{t,i},\alpha_i)=\frac{c_{t,i}^{1-\alpha_i}-1}{1-\alpha_i}, \tag{2b}$$

where I_t is the set of all the information available in period t, and $\alpha_i>0$ is the risk-

one-period optimization problem (as in CDM [2002]), where the middle-aged agent is the sole decision maker.

aversion parameter.

Our working assumption is that α_2 (risk aversion when old) is higher than α_1 (risk aversion when middle-aged).[7] This assumption is motivated by a large body of empirical work that has consistently documented an upward-sloping pattern of risk aversion over the life cycle. Some studies base their analysis on responses to survey questions designed to elicit individual risk preferences. For instance, Sung and Hanna (1996) analyze responses on risk tolerance from the Survey of Consumer Finances and find that risk tolerance decreases with age. Likewise, Dohmen et al. (2011) elicit risk attitudes using a set of survey questions and find that the proportion of individuals who are relatively unwilling to take risks increases strongly with age. Barsky et al. (1997) conclude that individuals between ages 55 and 70 are more risk intolerant than other age cohorts based on survey answers to risky scenarios.[8]

A number of other works investigate risk attitudes over the life cycle from observed portfolio allocation decisions. Morin and Suarez (1983) study the effect of age on households' demand for risky assets and conclude that risk aversion displays a distinct life-cycle pattern, increasing uniformly with age. Similarly, Bakshi and Chen (1994) use U.S. asset allocation data post-1945 and document a strong pattern of increasing risk aversion with age. Riley and Chow (1992) derive risk-aversion indices from actual asset allocation and find that risk aversion decreases with age until 65 and then increases significantly. A positive relation between age and risk aversion is also documented in a number of other studies that investigate household asset allocation choices (Palsson, 1996; Lee and Hanna, 1995). Finally, studies that focus on observed portfolio allocation decisions have consis-

[7] More broadly $\alpha_2 > \alpha_1 > \alpha_0$ where α_0 is the risk aversion of the young cohort. However, because the young do not participate in financial markets due to the borrowing constraint, the two relevant risk-aversion parameters in our model are α_1 and α_2. The borrowing constraint thus introduces some form of limited participation because agents participate in the market in two out of the three periods—as savers when middle-aged and as dissavers when old.

[8] A few studies have documented either a constant or a decreasing risk aversion with age, at least up until retirement (see, e.g., Wang and Hanna, 1997). Nonetheless, there seems to be a general agreement that risk aversion increases beyond age 65 (retirement age). This is corroborated by a drop in stock market participation rates and a decline in risky portfolio shares for agents older than 65.

tently reported strong life-cycle patterns for stock market participation and stock holdings: a hump-shaped participation profile over the life cycle, a decline in equity shares as investors approach retirement, and a stock market exit after retirement (Fagereng, Gottlieb, and Guiso, 2013; Guiso, Haliassos, and Jappelli, 2002).[9]

The rest of the model setup follows closely the Constantinides, Donaldson, and Mehra (2002) framework. Market clearing in period t requires that the demand for bonds and equity by the young and the middle-aged consumers equal their fixed supply.[10] Because the young effectively elect not to participate in the borrowing-constrained economy, the supply of bonds and equity must equal the demand of the middle-aged:

$$x_{t-1,1}^{b} = b \qquad \text{and} \qquad x_{t-1,1}^{e} = 1. \tag{3}$$

We model the joint process of aggregate income and wages of the middle-aged, (y_t, w_t^1), as a time-stationary probability distribution where the aggregate income y_t is given by $y_t = w^0 + w_t^1 + b + d_t$. In the calibration, y_t and w_t^1 assume two values each (representing good/bad states) for a total of four possible realizations (y_t, w_t^1) represented by four states ($s_t = j$, where $j = 1, ..., 4$). The 4×4 transition probability matrix is denoted by Π.

A stationary rational expectations equilibrium in this economy is a set of consumption/investment choices and a pair of bond and equity prices that maximize consumer expected utility $(2a) - (2b)$ and satisfy the market clearing conditions (3). Given the consumption constraints $(1a) - (1c)$, the consumer optimization problem with respect to

[9] It is also possible that older agents may appear to be more risk averse not because of an exogenous attitudinal change towards risky outcomes but because they face larger uncertainty over the remainder of their lifetime relative to other age cohorts (such as pension uncertainty or significantly larger health expenditures). In addition, long-horizon mean reversion in stock returns implies that equities may effectively appear to be riskier for the elderly given their relatively shorter investment horizon. These considerations may prompt the older agents to behave as if they are indeed more risk averse than other age cohorts. The age-dependent value of α_i can certainly be motivated by these factors in addition to the abundant empirical evidence cited above.

[10] Similar to CDM (2002), in our baseline case we abstain from growth and consider an economy that is stationary in levels. One limitation of this setup is that it assumes a fixed supply of assets (bonds and equity) over long period, which is, admittedly, somewhat unrealistic. There are a number of ways to remedy this issue: one that we explore in Section 4.3 is the introduction of exogenous growth.

$x^b_{t,1}$ and $x^e_{t,1}$ yields the following first-order conditions:

$$u'(c_{t,1})\, p^b_{t+1} = E(\beta u'(c_{t,2})\,(p^b_{t+2} + 1)|I_t) \tag{4a}$$

$$u'(c_{t,1})\, p^e_{t+1} = E(\beta u'(c_{t,2})\,(p^e_{t+2} + d_{t+2})|I_t). \tag{4b}$$

The share of wealth saved/invested by the middle-aged investor, and the relative shares of wealth in bonds and equity are

$$\Phi^s_{t,1} = \frac{x^b_{t,1}p^b_{t+1} + x^e_{t,1}p^e_{t+1}}{w^1_{t+1}}, \qquad \Phi^b_{t,1} = \frac{x^b_{t,1}p^b_{t+1}}{w^1_{t+1}}, \qquad \text{and} \qquad \Phi^e_{t,1} = \frac{x^e_{t,1}p^e_{t+1}}{w^1_{t+1}}, \tag{5}$$

where $\Phi^s_{t,1}$ denotes the total share of savings/investment as a proportion of the wage income of the middle-aged, while $\Phi^b_{t,1}$ and $\Phi^e_{t,1}$ denote the relative shares of wealth invested in bonds and equity, respectively. Likewise, the portfolio allocations of bonds and equity as a proportion of the financial portfolio are given by

$$\omega^b_{t,1} = \frac{x^b_{t,1}p^b_{t+1}}{x^b_{t,1}p^b_{t+1} + x^e_{t,1}p^e_{t+1}} \qquad \text{and} \qquad \omega^e_{t,1} = \frac{x^e_{t,1}p^e_{t+1}}{x^b_{t,1}p^b_{t+1} + x^e_{t,1}p^e_{t+1}}, \tag{6}$$

where $\omega^b_{t,1}$ and $\omega^e_{t,1}$ reflect the portfolio shares of bonds and equity, respectively.

Using market clearing conditions (3) and dropping the time subscripts, we can write $(4a) - (4b)$ as

$$u'(c_1)\, p^b(j) = \beta \sum_{k=1}^{4} (u'(c_2)\,\{p^b(k) + 1\})\Pi_{jk} \tag{7a}$$

$$u'(c_1)\, p^e(j) = \beta \sum_{k=1}^{4} (u'(c_2)\,\{p^e(k) + d(k)\})\Pi_{jk}, \tag{7b}$$

with

$$c_1 = w^1(j) - bp^b(j) - p^e(j) \tag{8a}$$

$$c_2 = b(p^b(j) + 1) + p^e(j) + d(j) \tag{8b}$$

for each state j of the economy. With IRA preferences, the marginal utilities of the middle-aged and old consumers are, respectively, $u'(c_1) = c_1^{-\alpha_1}$ and $u'(c_2) = c_2^{-\alpha_2}$. Substituting the dynamic budget constraint and the marginal utilities, equilibrium security prices can be written as

$$\frac{p^b(j)}{(w^1(j) - bp^b(j) - p^e(j))^{\alpha_1}} = \beta \sum_{k=1}^{4} \frac{\{p^b(k) + 1\}\Pi_{jk}}{(b(p^b(k) + 1) + p^e(k) + d(k))^{\alpha_2}} \tag{9a}$$

and

$$\frac{p^e(j)}{(w^1(j) - bp^b(j) - p^e(j))^{\alpha_1}} = \beta \sum_{k=1}^{4} \frac{\{p^e(k) + d(k)\}\Pi_{jk}}{(b(p^b(k) + 1) + p^e(k) + d(k))^{\alpha_2}}. \tag{9b}$$

These are the two equations to be estimated. Note that the price pairs $p^b(j)$ and $p^e(j)$ are functions of the two risk-aversion parameters (α_1 and α_2), which is a unique feature of the IRA preference setup.

2.2 Calibration

In order to focus exclusively on the effect of IRA preferences, our calibration parameters are identical to those in Constantinides, Donaldson, and Mehra (2002) and are reported in Table 1.[11] Here, we briefly outline the main parameters, referring the reader to the CDM (2002) study for a more detailed discussion. Further details on the calibration, the solution method, and its implementation are provided in the Appendix.

Note that one period in our model spans 20 years (one generation); however, all parameters (and results) are converted to annualized values and reported as such. Following CDM (2002), the transition matrix of the joint Markov process of the aggregate income and the wage of the middle-aged consumers, (y_t, w_t^1), is given by

$$\begin{bmatrix} & (y_1, w_1^1) & (y_1, w_2^1) & (y_2, w_1^1) & (y_2, w_2^1) \\ (y_1, w_1^1) & \phi & \pi & \sigma & H \\ (y_1, w_2^1) & \pi + \Delta & \phi - \Delta & H & \sigma \\ (y_2, w_1^1) & \sigma & H & \phi - \Delta & \pi + \Delta \\ (y_2, w_2^1) & H & \sigma & \pi & \phi \end{bmatrix},$$

where $\phi + \pi + \sigma + H = 1$. Nine parameters need to be determined: $y_1/E(y)$, $y_2/E(y)$, $w_1^1/E(y)$, $w_2^1/E(y)$, ϕ, π, σ, H, and Δ. Similar to CDM (2002), these parameters depend on a set of ratios, autocorrelations, and cross-correlations of the aggregate macroeconomic

[11] In this regard, our study is subject to the same criticisms and limitations as the original CDM (2002) model.

variables.[12] Specifically, they are derived from the following quantities (also summarized in Table 1): 1) the average share of income going to labor ($E(w^1 + w^0)/E(y)$), set at 0.65; 2) the average share of income going to the labor of the young, $w^0/E(y)$, set at 0.19; 3) the average share of income going to interest on government debt, $b/E(y)$, set at a historical 0.03; 4) the coefficient of variation of the 20-year wage income of the middle-aged, $\sigma(w^1)/E(w^1)$, fixed at 0.25; 5) the coefficient of variation of the 20-year aggregate income, $\sigma(y)/E(y)$, set at 0.20; 6) the 20-year autocorrelation of middle-aged wages, $corr(w_t^1, w_{t-1}^1)$, set at 0.1; 7) the 20-year autocorrelation of aggregate income, $corr(y_t, y_{t-1})$, set at 0.1; and 8) the 20-year cross-correlation of aggregate income and middle-aged wages, $corr(y_t, w_t^1)$, set at 0.1.[13]

Table 2, Panel A (from CDM [2002]), shows the historical mean and standard deviations of the annualized, 20-year holding-period return on the S&P 500 series and on the Ibbotson U.S. Government Treasury Long-Term bond yield. As seen, the real mean equity return is between 6% to 6.7%, the mean bond real return is around 1%, and the mean equity premium (that we seek to match) is between 5.3% and 6.6%.

Panel B summarizes estimates of the share of the risky asset in optimal portfolios from a number of studies in the literature. Bertaut and Starr-McCluer (2002) estimate the share of the risky asset in the portfolio to be around 54.4%. Similarly, Gomes and Michaelides (2005) estimate the average equity holdings as a share of financial wealth to be 54.8%. Other studies document a pronounced relation between age and the share of equity in portfolios. Guiso, Haliassos, and Jappelli (2002) show a hump-shaped pattern for risky asset holdings by age: the share of risky asset in the portfolio is around 34% for individuals less than 30 years old, rises to around 56% for individuals between the ages of 30 and 60, drops to below 50% for those between 60 and 69, and levels off to 33% for

[12] As discussed in Section 3, our model admits nonstationarity given that the stochastic discount factor depends on the relative difference $\alpha_1 - \alpha_2$ as well as on the scale of the economy (aggregate income). This differs from the CDM (2002) specification, where the homogeneity introduced by CRRA preferences results in equilibrium prices being stationary and scale independent. We address this issue in Section 4.2, where we present results for different scales of the economy.

[13] Results for other correlation structures are presented in Section 4.1.

individuals over 70. Ameriks and Zeldes (2004) also document a similar pattern with the share of the risky asset rising from 40% to 52% for ages 35 to 60 and declining after that. Because in our paper the middle-aged are the stockholding portion of the population, the share of the risky asset we seek to match is in the 40% to 55% range.

3. Results

3.1 Security returns and equity premium

The effect of IRA preferences on the security returns, equity premium, savings, and portfolio shares are summarized in Tables 3–8. We present results for three different levels of risk aversion (2, 4, and 6), calibrating a total of 18 model economies.

As a preliminary step and in order to better highlight the role of IRA preferences, we report results for the CRRA model of CDM (2002) with borrowing constraints (Table 3, Column (1)). A couple of familiar results stand out. First, as the average level of risk aversion in the economy increases, equity returns rise, bond returns decline, and the equity premium increases. This is expected because more risk-averse investors require higher equity returns given equity's risk and demand more bonds (suppressing equilibrium bond returns), with both effects working toward increasing the equity premium. Second, and more importantly, with CRRA preferences the equity premium is still short of its historical average even when risk aversion is relatively high ($\alpha = 6$), despite the borrowing constraint.

These results change dramatically with the introduction of IRA preferences, leading to results that are more in line with the empirical evidence. As summarized in Table 3, for each level of risk aversion, IRA preferences lead to an increase in both equity and bond returns, and a higher equity premium. The higher premium obtains because as agents become more risk averse over the life cycle ($\alpha_2 > \alpha_1$), they demand higher returns on both equity and bonds relative to CRRA utility. Because equity is riskier than bonds, the increase in equity return is significantly larger than that of bonds, resulting in a higher premium. These results are even more pronounced when the individual becomes very risk-averse over the life cycle (i.e., the larger the differential $\alpha_1 - \alpha_2$).

Overall, the model delivers equity premium values that are consistent with their historical averages even for relatively low levels of risk aversion. For example, when comparing the CRRA case $\{\alpha_1, \alpha_2\} = \{4.00, 4.00\}$ to IRA pair $\{4.00, 4.25\}$, equity premium rises from 4.0% to 6.3% (Table 3, Panel B). It is important to note that even a small amount of risk-aversion heterogeneity delivers significant results—that is, the higher equity premium is obtained for fairly small differences in risk-aversion values over the life cycle.

These results are generated by an equilibrium mechanism that amplifies consumption volatility for the marginal investor (middle-aged agent) through two channels: a savings effect and a portfolio substitution effect. Broadly speaking, the introduction of IRA preferences decreases savings and tilts portfolio holdings away from equity and toward bonds. We examine these two effects in more detail next.

3.2 Savings and portfolio shares

The impact of IRA preferences on aggregate savings can be illustrated by analyzing the stochastic discount factor (SDF) in the IRA economy:

$$m_{t+1} = \frac{\beta c_{t,2}^{-\alpha_2}}{c_{t,1}^{-\alpha_1}} = \beta \left(\frac{c_{t,2}}{c_{t,1}} \right)^{-\alpha_1} c_{t,2}^{(\alpha_1 - \alpha_2)}. \tag{11}$$

The first term of (11), $\beta \left(\frac{c_{t,2}}{c_{t,1}} \right)^{-\alpha_1}$, denotes the standard SDF with CRRA preferences, while the term $c_{t,2}^{(\alpha_1-\alpha_2)}$ is due to the presence of age-dependent increasing risk aversion. This is an additional source of volatility that is fundamentally different from the pure aggregate consumption uncertainty. Because in our model middle-aged investors become more risk averse as they age ($\alpha_1 < \alpha_2$), the new term decreases the standard SDF, implying that agents are now less willing to shift consumption over time. In other words, the (middle-aged) agents anticipating that they will become even more averse to gambles that play out in the future (when old) save less and consume more relative to the CRRA scenario.

These insights are further highlighted in Tables 4 and 5. Table 4 presents consumption and savings patterns in all four states for CRRA (Panel A) and IRA preferences. First, in line with the discussion above, the variance of the middle-aged consumption is much higher with IRA preferences relative to the CRRA economy.[14] Second, the level of savings decreases as the middle-aged investors are now even less willing to give up some of their current consumption.

Table 5 further examines the pattern of savings behavior as the differential in risk aversion increases over the life cycle. Broadly speaking, there are two opposing forces that determine the level of savings/investment: while more risk-averse agents optimally prefer to invest less in risky assets, they are also more prudent and want to accumulate more wealth over the life cycle. With IRA preferences, the risk-aversion effect dominates the wealth accumulation effect, whereas the opposite is true with CRRA utility.[15]

The portfolio substitution effect occurs because while IRA agents optimally choose to save/invest less, they do so by reducing equity holdings substantially more than their bond holdings. In other words, age-dependent IRA causes not only a reduction in aggregate of savings, but also a substitution away from (risky) equity investment toward (safer) bonds investment. This is clearly illustrated in Table 6 where, for all the risk-aversion levels, as agents become more risk averse over the life cycle, they reduce the share of wealth invested in equity substantially while reducing the overall investment in bonds only marginally. The overall effect is an increase in bonds share and a decrease in equity share in the optimal portfolio. As expected, these results are significantly more pronounced when the risk-aversion profile over the life cycle is steep (i.e., the greater the

[14] The consumption of the old-age cohort is also quite variable under both preference specifications, leading the middle-aged to invest some of their wealth in bonds because bonds are a hedge against future consumption variability.

[15] With CRRA preferences, the level of aggregate savings increases marginally as the average risk aversion in the economy rises (Table 5, Column (1)).

difference between α_2 and α_1).

The savings and portfolio substitution effects tend to work in the same direction in the case of equity, delivering higher equity returns than CRRA utility. Lower aggregate savings imply lower demand for both equity and bonds, which raises required returns for both securities. At the same time, rebalancing of portfolio shares away from equity as agents become more risk averse further reinforces the savings effect on equity, delivering even higher equity returns. The opposite is true for bonds: lower aggregate savings tend to increase bonds returns while the portfolio substitution effect suppresses them (as demand for bonds increases). On balance, bond returns increase marginally.[16] However, because equity returns increase substantially more, equity premium also rises.

Importantly, IRA preferences deliver results that align more closely with historical data not only for security returns but also for savings and portfolio allocation shares. With CRRA preferences, savings as a share of income, Φ^s, is 28% (Table 5, Panel B), far above the 10% to 22% observed in the U.S. data. In contrast, the share of savings with IRA preferences is a more realistic 13% to 24%. Likewise, portfolio shares with CRRA utility are much more heavily skewed toward equity (62%) than what the data suggest (40%–55%). IRA preferences, on the other hand, deliver portfolio shares in the range of 45% to 60%, matching more closely the historical figures (Table 6, Panel B).

3.3 IRA preferences and the borrowing constraints

We find that the type of preference heterogeneity introduced in this paper—age-dependent IRA—has a complex interaction with the borrowing constraint. In the case of equity, IRA preferences reinforce the effect of the borrowing constraint, thus producing even higher equity returns. Equity returns are higher in a borrowing-constrained economy (relative to an unconstrained economy) because prices are driven exclusively by the middle-aged

[16] This is an undesirable feature of the model as it exacerbates the risk-free puzzle. Separating time and risk preferences is one common way to address this issue. DaSilva and Farka (2018) introduce Epstein-Zin preferences in the OLG model of CDM (2002) and find that bond returns are lower under this alternative specification.

agents for whom equity does not have as much appeal because consumption at this stage of the life cycle is highly correlated with equity income. IRA preferences further reinforce this effect: the marginal (middle-aged) investor faces a steeper risk-aversion profile over his life cycle, so he saves less and tilts portfolio shares away from equity and toward bonds. As discussed above, both effects work toward producing higher equity returns.

In contrast, IRA preferences impact bond returns differently from the borrowing constraint. The imposition of the constraint lowers bond returns in equilibrium because the young are not able to smooth their lifetime consumption by borrowing at the risk-free rate to invest in equity. With IRA preferences, as discussed above, the savings and portfolio substitution effects work in opposite directions on bonds, resulting in overall marginally higher bond returns.

3.4 Risk-aversion differential over the life cycle

The heterogeneity in risk aversion over the life cycle presented here is vital important to delivering model predictions that are more closely aligned with empirical data. The additional source of variation captured by the differential of the two risk-aversion parameters, $\alpha_1 - \alpha_2$, adds nontrivially to the volatility of the pricing kernel. In fact, our results strongly suggest that the "life-cycle outlook variation" (the steeper risk-aversion profile captured by $\alpha_1 - \alpha_2$) has a significantly more pronounced impact on asset prices than the average level of risk aversion in the economy. Put it differently, the difference between the two risk-aversion parameters (how much more risk averse old agents are relative to the middle-aged) matters more rather than the average risk aversion in the economy (how much more risk averse both cohorts are).

Table 7, Panel A illustrates this point: security returns and equity premium are higher with risk pair $\{\alpha_1, \alpha_2\} = \{2.00, 2.50\}$ than with $\{\alpha_1, \alpha_2\} = \{6.00, 6.25\}$ even though the average level of risk aversion is much higher in the second case. This is attributable to the fact that the risk-aversion profile (both in absolute and relative terms) is much more disparate under $\{2.00, 2.50\}$ scenario than under $\{6.00, 6.25\}$. In effect, although the average level of risk aversion is lower at $\{2.00, 2.50\}$, the life-cycle risk perception is much higher. As such, the level of aggregate saving is lower and portfolio holdings are skewed more heavily toward bonds than equity under this scenario relative to an economy with higher average risk aversion.

A closer look at the results reveals that both the absolute and the relative differences between α_2 and α_1 matter. Table 7, Panel B presents a set of cases when α_1 and α_2 are progressively increased, while maintaining the same absolute differential $\alpha_2 - \alpha_1 = 0.25$. Here again, the familiar results reappear: aggregate savings are significantly higher under $\{6.00, 6.25\}$ compared to other risk pairs because on a relative basis, the life-cycle risk-aversion profile is significantly flatter in this scenario ($\frac{\alpha_2}{\alpha_1} = 1.042$) when compared to, say, $\{2.00, 2.25\}$ ($\frac{\alpha_2}{\alpha_1} = 1.125$).[17] In contrast, when maintaining a constant relative differential in risk aversion ($\frac{\alpha_2}{\alpha_1} = 1.05$) as shown in Table 7, Panel C, savings are lower with risk pair $\{6.00, 6.30\}$ than with $\{2.00, 2.10\}$ because the differential in risk aversion is larger in absolute terms under the first scenario. This increases the volatility of the equilibrium intertemporal rate of marginal substitution (IRMS) for the middle-aged, resulting in higher equity premium.

3.5 IRA preferences and related literature

IRA preferences allow for changes in the risk-aversion profile of the agent, providing an additional source of volatility in the stochastic discount factor, allowing us to employ almost trivially low coefficients of risk aversion. Other models also use changing risk aversion to solve the equity premium puzzle. A widely popular model is habit formation, which increases the effective risk aversion by postulating that agents are averse to variations in habit-adjusted consumption rather than to consumption variation itself (Constantinides, 1990; Campbell and Cochrane, 1999).

[17] The additional savings occur through both equity and bonds (suppressing both returns), but because the majority of savings happens through bonds, bond returns decline significantly more and equity premium rises.

Similar to our model, standard habit preferences also exhibit savings and portfolio substitution effects, albeit the mechanism is fundamentally different from ours. With habit, households prevent their consumption from fluctuating too much by increasing precautionary savings and holding more bonds in the optimal portfolio. In a sense, habit formation is observationally equivalent to becoming more risk averse: agents pay a large amount to avoid consumption gambles even if the relative risk aversion is small as long as the habit parameter is relatively large. However, the degree of consumption smoothing with habit preferences is high enough to prevent them from generating large fluctuation in marginal utilities: variations in IRMS are entirely consumption-driven. With no additional sources of risk, it is not surprising that habit models generally require high levels of effective risk aversion to achieve desired results. In both these respects (higher savings and lower variation in IRMS), habit models differ fundamentally from the one considered here.

There is, however, an interesting overlap between our results and the habit framework of DaSilva, Farka, and Giannikos (2011). In their work, they examine the impact of two habit parameters (middle-aged, and old) in the same OLG framework of CDM (2002) and report complex interactions between the two-habit setup and asset price dynamics. Somewhat analogous to our work here, they also find that an increase in the habit persistence of the middle-aged decreases aggregate savings (because habit increases the agent's marginal utility leading to higher current consumption) and tilts investment away from equity into bonds (as the effective risk aversion rises).[18] These similarities point to a deeper connection between the two frameworks: both IRA and habit formation increase the risk-aversion profile of the middle-aged agent over the life cycle.[19]

[18]The IRMS of DaSilva, Farka, and Giannikos (2011) model is given by $IRMS = \frac{\beta E\left[u'(c_{t,2}-\eta_2 c_{t,1})\right]}{u'(c_{t,1}-\eta_1 c_{t,0})-\eta_2 \beta E[u'(c_{t,2}-\eta_2 c_{t,1})]}$ where η_1 is the habit parameter for middle-aged and η_2 for old agents. Setting $\eta_2 = 0$, this simplifies to: $IRMS = \frac{\beta E\left[u'(c_{t,2})\right]}{u'(c_{t,1}-\eta_1 c_{t,0})} = \beta\left(\frac{c_{t,2}}{c_{t,1}-\eta_1 c_{t,0}}\right)^{-\alpha_1}$. It is easy to see that η_1 decreases the SDF, implying that agents are less willing to shift consumption over time.

[19]One drawback of DaSilva, Farka, and Giannikos (2011) is that they are able to match only around half of the historical premium.

Last, our IRA setup bears some similarities (at least in spirit) to the state dependent framework of Danthine et al. (2004). The authors explore a nontrivial generalization of the Mehra-Prescott model by allowing the coefficient of risk aversion to vary with the growth rate of consumption. Similar to our setup, this specification introduces another source of variation in the pricing kernel, which admits nonstationary asset returns as the SDF depends not only on the growth rate of consumption but also on its level ($IRMS_{t,t+1} = \beta \left(\frac{1}{x_{t+1}}\right)^{\alpha_{t+1}} y_t^{\alpha_t - \alpha_{t+1}}$, where x_{t+1} is consumption growth rate). $\alpha_t - \alpha_{t+1}$ represents a new source of uncertainty in the model, one that the authors term "mood" or "outlook" uncertainty. However, this term appears to induce extreme variations in IRMS as $\alpha_t - \alpha_{t+1}$ is stochastically negative and then positive. As a result, the effect of nonstationarity is amplified and the model delivers equity returns that are too high, a risk-free rate that is asymptotically too low, and security volatilities that dramatically exceed the data.

4. Model extensions and robustness

In this section, we evaluate the robustness of our baseline findings by specifying economies that incorporate different correlation structures, calibrated scales, exogenous growth, and pension schemes. We find that overall, our baseline results are robust to these alternative specifications and that the central message of this paper—that IRA preferences improve the outcomes of a standard OLG model with borrowing constraints—remains essentially unchanged.

4.1 Different correlation structures

In our baseline results, the 20-year cross-correlation of aggregate income and middle-aged wages, the 20-year autocorrelation of aggregate income, and the 20-year autocorrelation of middle-aged wages are set at a relatively low level ($corr(y_t, w_t) = corr(y_t, y_{t-1}) = corr(w_t^1, w_{t-1}^1) = 0.1$). However, because we lack sufficient time-series data to fully estimate these cross-correlation and autocorrelations, we follow CDM (2002) and present results for a variety of correlation structures: $corr(y_t, w_t) = 0.1$ and $corr(y_t, y_{t-1}) = corr(w_t^1, w_{t-1}^1) = 0.8$; $corr(y_t, w_t) = 0.8$ and $corr(y_t, y_{t-1}) = corr(w_t^1, w_{t-1}^1) = 0.1$; and $corr(y_t, w_t) = 0.8$ and $corr(y_t, y_{t-1}) = corr(w_t^1, w_{t-1}^1) = 0.8$).

Results are reported in Table 8. Our baseline findings remain generally unchanged: the premium is somewhat higher under some correlation structures and lower with others, but these differences are not large, attesting to the robustness of the model along this dimension.

4.2 Scale effects

As discussed above, a potential issue arising from IRA preferences is that the SDF depends on the scale of the economy (i.e., the level of consumption), leading to nonstationarity issues. As noted in Section 3, the SDF is given by

$$m_{t+1} = \frac{\beta c_{t,2}^{-\alpha_2}}{c_{t,1}^{-\alpha_1}} = \beta \left(\frac{c_{t,2}}{c_{t,1}} \right)^{-\alpha_1} c_{t,2}^{(\alpha_1 - \alpha_2)},$$

where $c_{t,2}^{(\alpha_1-\alpha_2)}$ is due to IRA specification. Because the SDF depends on the level of consumption, asset returns and equity premium are no longer insensitive to scale specifications with IRA preferences as they are with CRRA.

To check for the sensitivity of our results with respect to scale, we calibrate our economies based on alternative levels of aggregate income, $E(y)$: the average level of income is doubled and then tripled. The results, presented in Table 9, are shown for various risk-aversion pairs (representing the same relative increase in risk aversion, $\frac{\alpha_2}{\alpha_1} = 1.05$) and are contrasted against the baseline case (Table 9, Column (1)).

Our findings remain essentially unchanged under this modification. For example, for the pair $\{\alpha_1, \alpha_2\} = \{4.00, 4.20\}$, equity return increases by 0.30% when the scale doubles and by 0.48% when it triples, bond return increases by 0.16% and by 0.26%, respectively, and the equity premium increases by only 0.14% and 0.22%. In fact, doubling or tripling the scale of the economy results in only a marginal change (less than 5%) of the baseline results for all risk-aversion pairs.

4.3 Growth effects

In our baseline model, we follow CDM (2002) and abstract from growth, thereby considering an economy that is stationary in levels. This is in contrast to Mehra and Prescott (1985) who model an economy that is stationary in growth rates and has a unit root in levels.[20] Our decision to abstract from growth is motivated by the intent to stay as close as possible to the original CDM (2002) framework: our primary focus is in evaluating the importance of preference heterogeneity (as captured by IRA over the life cycle) on equity premium. Nonetheless, modern economies exhibit secular growth, which tends to increase the mean returns of financial assets relative to the no-growth alternative, even though real rates of return tend to be stationary. To evaluate the robustness of our results, we specify an analogous model that incorporates growth in our baseline IRA OLG model with borrowing constraints.[21]

Broadly speaking, the impact of growth on financial assets can be separated into two channels: the windfall effect and the portfolio substitution effect. The windfall effect arises because growth creates a preordained increase in future consumption. Investors, who are fully aware of this future windfall, require a greater return on all securities in order to save by postponing current consumption. The portfolio substitution effect arises from the fact that with growth, the share of output going to the wages of the young increases, while the value of dividends decreases. This means that equity becomes relatively less attractive when compared to the bond, which continues to pay a fixed coupon of one unit of consumption good in every period in perpetuity. Thus, the substitution effect tends to increase equity returns and decrease bond returns.

The overall effect of growth on financial assets therefore differs across securities: for

[20] As CDM (2002) point out, the choice of a stationary-in-levels economy is partially motivated by the fact that the model ends up being computationally simpler than a growth-stationary economy, and partially because a no-growth economy is consistent with the zero population growth feature of the model.

[21] We assume that output and population grow exogenously at a deterministic rate. A more complete model would require specifying a stochastic growth process, which can be taken up by future research and is outside the scope of the simple extension considered here.

equity, the substitution effect magnifies the windfall effect resulting in higher equity returns compared to the no-growth setup. For the bond, the substitution effect works in the opposite direction to the windfall effect, which means that the overall effect on bond returns is unknown a priori.

The results from this model generalization are presented in Table 10. We compare the baseline case ($n = 0$) with a growth rate of $n = 2\%$ for a number of risk-aversion pairs. As expected, with secular growth, equity returns increase as both the windfall effect and substitution effect work in the same direction. Bond returns decline for all our risk pairs, suggesting that the substitution effect dominates the windfall effect. The increase in equity returns and the decline in bond returns combine for a rise in equity premium. In addition, when compared to the no-growth scenario, growth effects tend to be more pronounced in economies with a higher average level of risk aversion.

4.4 ***Pension benefits***

As in Constantinides, Donaldson, and Mehra (2002), our baseline model assumes that consumers receive zero wage income when old. This assumption can be relaxed to allow for pension income and social security benefits. The introduction of pension benefits may have implications for our baseline findings because pension income affects savings, security returns, and equity premium (see, among others, Abel [2003], Bohn [1999], Campbell and Nosbusch [2007], and Olovsson [2004]). This extension is also motivated by the fact that the historical value of the equity premium in the United States appears to be substantially higher since the introduction of the current U.S. (pay-as-you-go [PAYGO]) Social Security system in 1935: Mehra and Prescott [2003] document the equity premium for the United States to be 3.92% from 1889 to 1933 and 8.93% from 1934 to 2000.

While social security income reduces the need for precautionary savings (because of guaranteed retirement income), its impact on the risk premium is theoretically ambiguous. Broadly speaking, the introduction of a pension scheme affects equity and bond returns through two channels: the implicit-asset channel and the income channel. The implicit-asset effect increases bond returns and lowers equity returns, thus decreasing the equity premium obtained in equilibrium. The mechanism is intuitively straightforward: in the presence of pension income, investors effectively hold an implicit second asset (the claim to future social security benefits), which is a relatively safe asset and as such exhibits bond-like features. This reduces the need to hold bonds directly in the portfolio (which increases bond returns) while increasing the demand for equity (reducing equity returns), resulting in a lower premium. In other words, the presence of social security income effectively makes investors less averse to equity risk, thereby reducing the risk premium they require in equilibrium.

The income effect works in the opposite direction for equity (increasing equity returns) while reinforcing the implicit asset effect for bonds (further increasing bond returns). With deterministic pension income, investors must be paid a higher rate of return (in both equity and bonds) to entice them to save. This leads to an increase in both equity and bond returns. Thus, the overall impact of pension on the equity premium is ambiguous.[22]

Our model extension considers a pension scheme similar to the U.S. (PAYGO) Social Security system where benefits to current retirees are financed through taxes on those who work. As in the baseline scenario, consumers born in period t receive (low) deterministic $w^0 > 0$ wage income in period t and stochastic wage income w^1_{t+1} when middle-aged (in period $t+1$). However, consumers now receive a fixed social security benefit ss^2_{t+2} when old. To finance these benefits, a payroll tax rate, τ_t, is set on the young and the middle-aged (the currently working generations) so that their payroll tax contributions equal the exogenous benefits—that is,

[22] Campbell and Nosbusch (2007) argue that while social security income raises the average return on risky assets, it also increases the return volatility, which in turn forces investors to require a higher premium. Return volatility increases in our specification with pension income, providing an additional explanation for the higher risk premium we find with this generalization.

$$\tau_t = \frac{ss^2_{t+2}}{w^0 + w^1_{t+1}}. \tag{12}$$

With this payroll tax and social security benefits, the budget constraint for the consumer born in period t is now:

$$c_{t,0} \leq w^0(1-\tau_t) \qquad \text{when young} \tag{13a}$$

$$c_{t,1} \leq w^1_{t+1}(1-\tau_t) - x^b_{t,1}p^b_{t+1} - x^e_{t,1}p^e_{t+1} \qquad \text{when middle-aged} \tag{13b}$$

$$c_{t,2} \leq x^b_{t,1}(p^b_{t+2}+1) + x^e_{t,1}(p^e_{t+2}+d_{t+2}) + ss^2_{t+2} \qquad \text{when old.} \tag{13c}$$

Table 11 summarizes results from this specification. We consider different payroll tax rates when calibrating our results: the current U.S. payroll tax rate (12.4%), a lower rate (6.4%), and a higher rate (15%).[23] As expected, both the implicit-asset effect and the income effect work toward increasing bond returns. This means that the introduction of pension benefits tends to exacerbate the risk-free puzzle. Equity returns also rise, indicating that the income effect dominates the implicit-asset effect. Because equity is riskier than bonds, the required equilibrium equity return increases by more than the bond return delivering a higher equity premium.

Overall, our IRA results are fairly robust to the model generalization with pension income: equity premium increases modestly, from 5.8% with no pension income to 6.6% with pension income (with the tax rate set at the current 12.4%). Likewise, portfolio allocation shares do not vary much: the share of equity drops marginally from 48.9% to 43.9%, while the share invested in bonds increases modestly.

5. Conclusions

This paper addresses long-standing issues in the asset pricing literature, with focus on the equity premium puzzle. The novelty of the work lies in introducing preference heterogeneity over the life cycle. More specifically, we introduce age-dependent IRA into the

[23] The payroll tax rates of {0%, 6.4%, 12.4%, 15%} correspond to fixed social security benefit levels of $ss = \{\$0, \$3,936, \$7,872, \$9,447\}$.

overlapping generations economy of Constantinides, Donaldson, and Mehra (2002) with borrowing constraints.

We find that IRA preferences have profound implications for security returns and equity premium. Essentially, IRA specification introduces an additional source of uncertainty in the pricing kernel, which amplifies the volatility of consumption leading to large fluctuations in the marginal utilities. The mechanism works through the interplay of two main channels: a savings effect and a portfolio substitution effect. IRA agents save less (because they are more averse to future gambles) and rebalance portfolio holdings away from equity and toward bonds. Both effects cause an increase in equity returns because equity is now even less attractive relative to the bond. In contrast, the effect on bond returns is theoretically ambiguous: lower aggregate savings increases bond returns, while portfolio rebalancing tends to suppress them. On balance, bond returns increase marginally for the range of parameters considered in the model. Because equity returns increase substantially more, equity premium also rises.

The model generates results that are generally more consistent with U.S. data without assuming unreasonable levels of risk aversion. Strikingly, we find that a small amount of preference heterogeneity goes a long way toward improving the fit of the model, as our results are obtained with fairly small differences in risk-aversion parameters over the life cycle. This life-cycle outlook variation is vastly more important than the average level of risk aversion in the economy: an economy with low average risk aversion but large differential in lifetime risk-aversion profile is able to deliver a higher premium than an economy with a higher average risk aversion but flatter life-cycle risk-aversion profile.

The model is kept intentionally close to the baseline CDM (2002) framework in order to more starkly highlight the role of IRA preferences. As such, the model abstracts from some key features that may enrich its results, which we defer for future work. For example, the lack of labor income risk eliminates the precautionary savings motive, which would add more realism to the model. In addition, one limitation of IRA preferences is that it exacerbates the risk-free rate puzzle. One interesting generalization would be to introduce uninsurable labor income shocks or a fixed cost of stock market participation in this framework. Alternatively, the relaxation of the borrowing constraints may highlight more fully the role of IRA on the level of savings, household portfolio allocation, and security returns.

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Appendix: Details on calibration and model solution

Our calibration parameters are set as in Constantinides, Donaldson, and Mehra (2002). The equilibrium security prices are linear scalar multiples of wage and income variables and depend on the set of moment conditions $\frac{E(w^1+w^0)}{E(y)}$, $\frac{w^0}{E(y)}$, $\frac{b}{E(y)}$, $\frac{\sigma(w^1)}{E(w^1)}$, $\frac{\sigma(y)}{E(y)}$, $corr(y_t, w_t^1)$, $corr(y_t, y_{t-1})$, and $corr(w_t^1, w_{t-1}^1)$, which we proceed to calibrate as in CDM (2002).

We set the average share of income going to labor, $\frac{E(w^1+w^0)}{E(y)}$, equal to 0.65. Historically, the ratio has fluctuated between 0.65 and 0.75. As in CDM (2002), we set this ratio to the lower bound reflecting the fact that for the stockholding class (the middle-aged), the share of income going to labor will likely be lower than for the overall population. We follow CDM (2002) and set the share of income going to the young, $\frac{w^0}{E(y)}$, at a sufficiently low value of 0.19 so that the borrowing constraint is binding. Given our choice of the aggregate income (i.e., $E(y) = 98,399$), this means that the young consumer receives a low deterministic wage income (w^0) of \$19,000. The average share of income going to interest on the debt, $\frac{b}{E(y)}$, is set at 0.03, consistent with U.S. data. The coefficient of variation of the 20-year wage income of the middle-aged, $\frac{\sigma(w^1)}{E(w^1)}$, is set at 0.25 in line with studies such as Cox (1984), and Creedy (1985). The coefficient of variation of the 20-year aggregate income, $\frac{\sigma(y)}{E(y)}$, is set at 0.2, consistent with historical data using a detrended Hodrick-Prescott filter for aggregate income. Following CDM (2002), we set the 20-year autocorrelation of middle-aged wages, $corr(w_t^1, w_{t-1}^1)$; the 20-year autocorrelation of aggregate income, $corr(y_t, y_{t-1})$; and the 20-year cross-correlation of aggregate income and middle-aged wages, $corr(y_t, w_t^1)$, at 0.1.

The joint probability matrix, Π, is given by

$$\begin{bmatrix} & (y_1, w_1^1) & (y_1, w_2^1) & (y_2, w_1^1) & (y_2, w_2^1) \\ (y_1, w_1^1) & \phi & \pi & \sigma & H \\ (y_1, w_2^1) & \pi+\Delta & \phi-\Delta & H & \sigma \\ (y_2, w_1^1) & \sigma & H & \phi-\Delta & \pi+\Delta \\ (y_2, w_2^1) & H & \sigma & \pi & \phi \end{bmatrix}.$$

The transition matrix entries (ϕ, π, σ, H, and Δ) as well as $y_1/E(y)$, $y_2/E(y)$, $w_1^1/E(y)$, and $w_2^1/E(y)$ are chosen to satisfy the above moment conditions as well as the property that the probabilities add up to 1 ($\phi + \pi + \sigma + H = 1$). Writing the moment conditions in terms of the transition matrix yields the following transition probability matrix

$$\Pi = \begin{bmatrix} 0.5298 & 0.0202 & 0.0247 & 0.4253 \\ 0.0302 & 0.5198 & 0.4253 & 0.0247 \\ 0.0247 & 0.4253 & 0.5198 & 0.0302 \\ 0.4253 & 0.0247 & 0.0202 & 0.5298 \end{bmatrix},$$

as well as $y_1/E(y) = 1.20$, $y_2/E(y) = 0.80$, $w_1^1/E(y) = 0.57$, and $w_2^1/E(y) = 0.34$. The moment conditions are also used to compute the long-run stationary probability distribution $P = \begin{bmatrix} 0.275 & 0.225 & 0.225 & 0.275 \end{bmatrix}'$.

Once the values for ϕ, π, σ, H, Δ, $y_1/E(y)$, $y_2/E(y)$, $w_1^1/E(y)$, and $w_2^1/E(y)$ are obtained, they are used to solve for p^b and p^e (Equations $(9a) - (9b)$). We define

$$f(p^b(1)) = \frac{p^b(1)}{[w^1(1) - bp^b(1) - p^e(1)]^{\alpha_1}} - \beta \sum_{k=1}^{4} \frac{\{p^b(k) + 1\}\Pi_{jk}}{[b(p^b(k) + 1) + p^e(k) + d(k)]^{\alpha_2}}, \qquad \text{(A1)}$$

$$f(p^b(2)) = \frac{p^b(2)}{[w^1(2) - bp^b(2) - p^e(2)]^{\alpha_1}} - \beta \sum_{k=1}^{4} \frac{\{p^b(k) + 1\}\Pi_{jk}}{[b(p^b(k) + 1) + p^e(k) + d(k)]^{\alpha_2}}, \qquad \text{(A2)}$$

$$f(p^b(3)) = \frac{p^b(3)}{[w^1(3) - bp^b(3) - p^e(3)]^{\alpha_1}} - \beta \sum_{k=1}^{4} \frac{\{p^b(k) + 1\}\Pi_{jk}}{[b(p^b(k) + 1) + p^e(k) + d(k)]^{\alpha_2}}, \qquad \text{(A3)}$$

$$f(p^b(4)) = \frac{p^b(4)}{[w^1(4) - bp^b(4) - p^e(4)]^{\alpha_1}} - \beta \sum_{k=1}^{4} \frac{\{p^b(k) + 1\}\Pi_{jk}}{[b(p^b(k) + 1) + p^e(k) + d(k)]^{\alpha_2}}, \qquad \text{(A4)}$$

$$f(p^e(1)) = \frac{p^e(1)}{[w^1(1) - bp^b(1) - p^e(1)]^{\alpha_1}} - \beta \sum_{k=1}^{4} \frac{\{p^e(k) + d(k)\}\Pi_{jk}}{[b(p^b(k) + 1) + p^e(k) + d(k)]^{\alpha_2}}, \qquad \text{(A5)}$$

$$f(p^e(2)) = \frac{p^e(2)}{[w^1(2) - bp^b(2) - p^e(2)]^{\alpha_1}} - \beta \sum_{k=1}^{4} \frac{\{p^e(k) + d(k)\}\Pi_{jk}}{[b(p^b(k) + 1) + p^e(k) + d(k)]^{\alpha_2}}, \qquad \text{(A6)}$$

$$f(p^e(3)) = \frac{p^e(3)}{[w^1(3) - bp^b(3) - p^e(3)]^{\alpha_1}} - \beta \sum_{k=1}^{4} \frac{\{p^e(k) + d(k)\}\Pi_{jk}}{[b(p^b(k) + 1) + p^e(k) + d(k)]^{\alpha_2}}, \qquad \text{(A7)}$$

and

$$f(p^e(4)) = \frac{p^e(4)}{[w^1(4) - bp^b(4) - p^e(4)]^{\alpha_1}} - \beta \sum_{k=1}^{4} \frac{\{p^e(k) + d(k)\}\Pi_{jk}}{[b(p^b(k) + 1) + p^e(k) + d(k)]^{\alpha_2}}. \qquad \text{(A8)}$$

Let $F(x)$ be an 8×1 vector containing Equations $(A1) - (A8)$ above: $F(x) = \begin{bmatrix} f(p^b(1)) & f(p^b(2)) & f(p^b(3)) & f(p^b(4)) & f(p^e(1)) & f(p^e(2)) & f(p^e(3)) & f(p^e(4)) \end{bmatrix}'$.

Numerical solution for Equations $(A1) - (A8)$ is obtained by employing the Newton-Raphson method to solve for the root $r = \begin{bmatrix} p^b(1) & p^b(2) & p^b(3) & p^b(4) & p^e(1) & p^e(2) & p^e(3) & p^e(4) \end{bmatrix}'$: Starting with an initial estimate of $\mathbf{X}_0 = \mathbf{0}$ of the root r, our next estimate is $X_1 = X_0 - \frac{F(X_0)}{F'(X_0)} = 0 - \frac{F(0)}{F'(0)}$, where $F'(x)$ is an 8×8 matrix of the partial derivative of the price equations $F(x)$ with respect to the four bond and four equity prices (e.g., row 1 $\frac{\partial f(p^b(1))}{\partial p^b(1)}, \frac{\partial f(p^b(1))}{\partial p^b(2)}, \frac{\partial f(p^b(1))}{\partial p^b(3)}, \frac{\partial f(p^b(1))}{\partial p^b(4)}, \frac{\partial f(p^b(1))}{\partial p^b(1)}, \frac{\partial f(p^b(1))}{\partial p^e(1)}, \frac{\partial f(p^b(1))}{\partial p^e(2)}, \frac{\partial f(p^b(1))}{\partial p^e(3)}, \frac{\partial f(p^b(1))}{\partial p^e(4)}$) and so on. Following 500 successive iterations, the equilibrium bond and equity prices, $\{p^b(1), p^b(2), p^b(3), p^b(4)\}$ and $\{p^e(1), p^e(2), p^e(3), p^e(4)\}$ are obtained.

References

Abbot, T., 2017. Heterogeneous risk preferences in financial markets, Working paper, Institut d'Etudes Politiques de Paris (Sciences Po), Department of Economics.

Abel, A. B., 2003. The effects of a baby boom on stock prices and capital accumulation in the presence of social security, *Econometrica* 71, 551–578.

Ameriks, J., and S. Zeldes, 2004. How do household portfolio shares vary with age? Working paper, Columbia Business School.

Bakshi, G. S., and Z. Chen, 1994. Baby boom, population aging, and capital markets, *The Journal of Business* 67, 165–202.

Bansal, R., and A. Yaron, 2004. Risks for the long run: A potential resolution of asset pricing puzzles, *The Journal of Finance* 59, 1481–1509.

Barro, R., 2006. Rare disasters and asset markets in the twentieth century, *Quarterly Journal of Economics* 121, 832–866.

Barsky, R. B., T. F. Juster, M. S. Kimball, and M. D. Shapiro, 1997. Preference parameters and individual heterogeneity: An experimental approach in the health and retirement study, *Quarterly Journal of Economics* 112, 537–579.

Basak, S., and D. Cuoco, 1998. An equilibrium model with restricted stock market participation, *Review of Financial Studies* 11, 309–341.

Benzoni, L., P. Collin-Dufresne, and R. G. Goldstein, 2007. Portfolio choice over the life-cycle when the stock and labor markets are cointegrated, *Journal of Finance* 6, 2123–2167.

Bertraut, C., and M. Starr-McCluer, 2002. Household portfolios in the United States, in L. Guiso, M. Haliassos, and T. Jappelli, eds., Household Portfolios, MIT Press, Boston, MA.

Bhamra, H. S., and R. Uppal, 2014. Asset prices with heterogeneity in preferences and beliefs, *The Review of Financial Studies* 27, 519–580.

Bohn, H., 1999. Should the social security trust fund hold equities? An intergenerational welfare analysis, *Review of Economic Dynamics* 2, 666–697.

Campbell, J. Y., 1996. Understanding risk and return, *The Journal of Political Economy* 104, 298–345.

Campbell, J. Y., 2003. Consumption-based asset pricing, in G.M. Constantinides, M. Harris, and R.M. Stulz, eds., Handbook of the Economics of Finance, Elsevier.

Campbell, J. Y., and J. H. Cochrane, 1999. By force of habit: A consumption-based explanation of aggregate stock market behavior, *Journal of Political Economy* 107, 205–251.

Campbell, J. Y., and Y. Nosbusch, 2007. Intergenerational risksharing and equilibrium asset prices, *Journal of Monetary Economics* 54, 2251–2268.

Chan, Y. L., and L. Kogan, 2002. Catching up with the joneses: Heterogeneous preferences and the dynamics of asset prices, *Journal of Political Economy* 110, 1255–1285.

Cocco, J., F. Gomes, and P. Maenhout, 2005. Portfolio choice over the life cycle, *Review of Financial Studies* 18, 491–533.

Constantinides, G. M., 1990. Habit formation: A resolution of the equity premium puzzle, *Journal of Political Economy* 98, 519–543.

Constantinides, G. M., J. B. Donaldson, and R. Mehra, 2002. Junior can't borrow: A new perspective on the equity puzzle, *Quarterly Journal of Economics* 117, 269–296.

Cox, D., 1984. Inequality in the lifetime earnings of women, *Review of Economics and Statistics* 16, 501–504.

Creedy, J., 1985. Dynamics of Income Distribution, Basil Blackwell, New York, NY.

Danthine, J., J. B. Donaldson, C. Giannikos, and H. Guirguis, 2004. On the consequences of state dependent preferences for the pricing of financial assets, *Finance Research Letters* 1, 143–153.

DaSilva, A., and M. Farka, 2018. Asset pricing puzzles in an OLG economy with generalized preference, *European Financial Management* 24, 331–361.

DaSilva, A., M. Farka, and C. Giannikos, 2011. Habit formation in an overlapping generations model with borrowing constraints, *European Financial Management* 17, 705–725.

Dohmen, T., A. Falk, D. Huffman, U. Sunde, J. Schupp, and G.G. Wagner, 2011. Individual risk attitudes: Measurement, determinants, and behavioral consequences, *Journal of the European Economic Association* 9, 522–550.

Dumas, B., 1989. Two-person dynamic equilibrium in the capital market, *Review of Financial Studies* 2, 157–188.

Epstein, L., E. Farhi, and T. Strzalecki, 2014. How much would you pay to resolve long-run risk? *American Economic Review* 104, 2680–2697.

Epstein, L., and S. Zin, 1990. 'First-order' risk aversion and the equity premium puzzle, *Journal of Monetary Economics* 26, 387–407.

Fagereng, A., C. Gottlieb, and L. Guiso, 2013. Asset market participation and portfolio choice over the life-cycle, Working paper, European University Institute.

Gârleanu, N., and S. Panageas, 2015. Young, old, conservative and bold: The implications of heterogeneity and finite lives for asset pricing, *Journal of Political Economy* 123, 670–685.

Gomes, F., and A. Michaelides, 2005. Optimal life-cycle asset allocation: Understanding the empirical evidence, *Journal of Finance* 60, 869–904.

Gomes, F., and A. Michaelides, 2008. Asset pricing with limited risk sharing and heterogeneous agents, *Review of Financial Studies* 21, 415–448.

Guiso, L., M. Haliassos, and T. Jappelli, 2002. Household Portfolios, MIT Press, Boston, MA.

Guvenen, F., 2009. A parsimonious macroeconomic model for asset pricing, *Econometrica* 77, 1711–1750.

Heaton, J., and D. Lucas, 1997. Market frictions, savings behavior, and portfolio choice, *Macroeconomic Dynamics* 1, 76–101.

Kocherlakota, N., 1996. The equity premium: It's still a puzzle, *Journal of Economic Literature* 34, 42–71.

Kogan, L., I. Makarov, and R. Uppal, 2007. The equity risk premium and the risk-free rate in an economy with borrowing constraints, *Mathematics and Financial Economics* 1, 1–19.

Lee, H., and S. Hanna, 1995. Investment portfolios and human wealth, *Financial Counseling and Planning* 6, 147–152.

McGrattan, E., and E. C. Prescott, 2003. Average debt and equity returns: Puzzling? *American Economic Review Papers and Proceedings* 93, 392–397.

Mehra, R., and E. C. Prescott, 1985. The equity premium: A puzzle, *Journal of Monetary Economics* 15, 145–161.

Mehra, R., and E. C. Prescott, 2003. The equity premium in retrospect, in G. M. Constantinides, M. Harris, and R. M. Stulz, eds., Handbook of the Economics of Finance, Elsevier.

Melino, A., and A. X. Yang, 2003. State-dependent preferences can explain the equity premium puzzle, *Review of Economic Dynamics* 6, 806–830.

Morin, R., and A. F. Suarez, 1983. Risk aversion revisited, *The Journal of Finance* 38, 1201–1216.

Olovsson, C., 2004. Social security and the equity premium puzzle, Working paper 729, Institute for International Economic Studies.

Palsson, A-M., 1996. Does the degree of relative risk aversion vary with household characteristics? *Journal of Economic Psychology* 17, 771–787.

Rietz, T. A., 1988. The equity risk premium: A solution, *Journal of Monetary Economics* 22, 117–131.

Riley, W. B., and K. V. Chow, 1992. Asset allocation and individual risk aversion, *Financial Analysts Journal* 48, 32–37.

Shiller, R. J., 1981. Do stock prices move too much to be justified by subsequent changes in dividends? *American Economic Review* 71, 421–436.

Siegel, J. J., 1992. The equity premium: Stock and bond returns since 1802, *Financial Analysts Journal* 48, 28–38.

Storesletten, K., C. Telmer, and A. Yaron, 2007. Asset pricing with idiosyncratic risk and overlapping generations, *Review of Economic Dynamics* 10, 519–548.

Sung, J., and S. Hanna, 1996. Factors related to risk tolerance, *Financial Counseling and Planning* 7, 11–20.

Telmer, C., 1993. Asset-pricing puzzles and incomplete markets, *Journal of Finance* 48, 1803–1832.

Wang, H., and S. Hanna, 1997. Does risk tolerance decrease with age? *Financial Counseling and Planning* 8, 27–31.

Wang, J., 1996. The term structure of interest rates in a pure exchange economy with heterogeneous investors, *Journal of Financial Economics* 41, 75–110.

Weil, P., 1989. The equity premium puzzle and the risk-free rate puzzle, *Journal of Monetary Economics* 24, 401–421.

Weinbaum, D., 2012. Preference heterogeneity and asset prices: An exact solution, *Journal of Banking and Finance* 34, 2238–2246.

Xiouros, C., and F. Zapatero, 2010. The representative agent of an economy with external habit formation and heterogeneous risk-aversion, *Review of Financial Studies* 23, 3017–3047.

Table 1. Parameter Values Used in the Calibration

This table presents the list of all the parameters used in the calibration of the model.

Coefficient of Risk Aversion for Middle-Aged: $\boldsymbol{\alpha_1}$	2, 4, and 6
Coefficient of Risk Aversion for Old Agents: $\boldsymbol{\alpha_2}$	2.05...2.25; 4.05...4.25; 6.05…6.35
Subjective Discount Rate: $\boldsymbol{\beta}$ implies an annual $\beta = 0.96$	0.44
Average Aggregate Income: **E(y)**	\$98,399
Average Wage Income for the Middle-Aged: $\mathbf{E(w^1)}$	\$44,650
Average Share of Income going to Labor: $\mathbf{E(w^0+w^1)/E(y)}$	0.65
Average Share of Income going to the Young: $\mathbf{w^0/E(y)}$	0.19
Average Share of Income going to Government Debt: **b/(E(y)**	0.03
Coefficient of Variation of 20-year Wage Income: $\boldsymbol{\sigma}\mathbf{(w^1)/E(w^1)}$	0.25
Coefficient of Variation of 20-year Aggregate Income: $\boldsymbol{\sigma}\mathbf{(y)/E(y)}$	0.20
$Corr(y_t,w^1_t)$; $Corr(y_t,y_{t-1})$; $Corr(w^1_t,w^1_{t-1})$	0.10
$P_1 = 0.275$; $P_2 = 0.225$; $P_3 = 0.225$; $P_4 = 0.275$	

Table 2. Historical Security Returns, Equity Premium, and Equity Share in the Optimal Portfolio

Panel A is a replica of Table I in Constantinides, Donaldson, and Mehra (2002). It shows the mean and standard deviations of the annualized, 20-year holding-period return on the S&P 500 total return series and on the Ibbotson U.S. Government Treasury Long-Term bond yield. Real returns are CPI adjusted. The annualized mean return (for both the equity and bond) is defined as the sample mean of the $\log(20 - \textit{year holding period return})/20$. Annualized standard deviation of the equity (or bond) return is defined as the sample standard deviation of the $\log(20 - \textit{year holding period return})/\sqrt{20}$. The annualized mean equity premium is defined as the difference of the mean return on equity and the mean return on the bond. The standard deviation of the premium is defined as the sample standard deviation of the $[\log(20 - \textit{year nominal equity return}) - \log(20 - \textit{year nominal bond return})]/\sqrt{20}$.

Panel B shows the average share of the risky asset in the optimal portfolio as computed by a number of empirical studies.

Panel A. Historical U.S. Real Returns

	1/1889-12/1999			***1/1926-12/1999***		
	Equity	**Bond**	**Premium**	**Equity**	**Bond**	**Premium**
Mean	**6.15**	**0.82**	**5.34**	**6.71**	**0.14**	**6.58**
Standard Deviation	13.95	7.40	14.32	15.79	7.25	15.21

Panel B. Average Share of the Risky Asset in Portfolios

Bertaut and Starr-McCluer (2002)	54.4%
Gomes and Michaelides (2005)	54.8%
Ameriks and Zeldes (2004)	40%–52%
Guiso, Haliassos, and Jappelli (2002)	38%–56%

Table 3. Security Returns and Equity Premium: CRRA and IRA Preferences

The impact of IRA on security returns and on the equity premium, for different levels of middle-aged risk aversion (2, 4, and 6). Results are shown for both CRRA and IRA preferences. All statistics are reported in annualized percentage terms.

	CRRA	***IRA***				
	$\alpha_1 = 2.00$ $\alpha_2 = 2.00$	$\alpha_1 = 2.00$ $\alpha_2 = 2.05$	$\alpha_1 = 2.00$ $\alpha_2 = 2.10$	$\alpha_1 = 2.00$ $\alpha_2 = 2.15$	$\alpha_1 = 2.00$ $\alpha_2 = 2.20$	$\alpha_1 = 2.00$ $\alpha_2 = 2.25$
Mean Equity Return	**6.86%**	**7.29%**	**7.91%**	**8.61%**	**9.87%**	**10.97%**
St. Dev of Equity Return	16.42%	17.29%	18.43%	19.77%	21.23%	22.71%
Mean Bond Return	**4.73%**	**4.80%**	**4.78%**	**4.88%**	**5.50%**	**5.93%**
St. Dev of Bond Return	12.75%	13.22%	13.89%	14.73%	15.66%	16.64%
Mean Equity Premium	**2.13%**	**2.49%**	**3.13%**	**3.73%**	**4.37%**	**5.04%**
St. Dev of Equity Premium	16.24%	17.59%	19.09%	20.60%	21.95%	22.98%

	CRRA	***IRA***				
	$\alpha_1 = 4.00$ $\alpha_2 = 4.00$	$\alpha_1 = 4.00$ $\alpha_2 = 4.05$	$\alpha_1 = 4.00$ $\alpha_2 = 4.10$	$\alpha_1 = 4.00$ $\alpha_2 = 4.15$	$\alpha_1 = 4.00$ $\alpha_2 = 4.20$	$\alpha_1 = 4.00$ $\alpha_2 = 4.25$
Mean Equity Return	**7.95%**	**8.49%**	**9.03%**	**9.56%**	**10.10%**	**10.85%**
St. Dev of Equity Return	20.56%	20.96%	21.43%	21.94%	22.49%	23.10%
Mean Bond Return	**3.99%**	**4.13%**	**4.23%**	**4.30%**	**4.33%**	**4.52%**
St. Dev of Bond Return	17.21%	17.35%	17.58%	17.91%	18.31%	18.76%
Mean Equity Premium	**3.97%**	**4.36%**	**4.80%**	**5.26%**	**5.77%**	**6.33%**
St. Dev of Equity Premium	21.01%	21.49%	21.92%	22.23%	22.37%	22.28%

	CRRA	***IRA***				
	$\alpha_1 = 6.00$ $\alpha_2 = 6.00$	$\alpha_1 = 6.00$ $\alpha_2 = 6.05$	$\alpha_1 = 6.00$ $\alpha_2 = 6.10$	$\alpha_1 = 6.00$ $\alpha_2 = 6.15$	$\alpha_1 = 6.00$ $\alpha_2 = 6.20$	$\alpha_1 = 6.00$ $\alpha_2 = 6.25$
Mean Equity Return	**8.42%**	**8.76%**	**9.17%**	**9.64%**	**10.19%**	**10.78%**
St. Dev of Equity Return	23.04%	23.12%	23.26%	23.47%	23.75%	24.12%
Mean Bond Return	**3.75%**	**3.82%**	**3.91%**	**4.05%**	**4.22%**	**4.38%**
St. Dev of Bond Return	19.11%	19.13%	19.26%	19.48%	19.81%	20.23%
Mean Equity Premium	**4.67%**	**4.94%**	**5.26%**	**5.59%**	**5.97%**	**6.40%**
St. Dev of Equity Premium	21.70%	22.10%	22.45%	22.74%	22.93%	22.49%

Table 4. Consumption and Savings/Investment across Different States

Consumption, savings, equity and bond investments, and security returns across all four states with CRRA (Panel A) and IRA preferences (Panel B).

	***CRRA:* α_1 = 4.00 α_2 = 4.00**				
	State 1	**State 2**	**State 3**	**State 4**	**Average**
Middle-Aged Consumption	**$38,768**	**$34,430**	**$26,821**	**$27,979**	**$32,137**
Old Consumption	**$60,432**	**$25,168**	**$72,379**	**$31,619**	**$47,262**
Young Consumption	$19,000	$19,000	$19,000	$19,000	$19,000
Savings/Investment	**$17,082**	**$21,420**	**$6,629**	**$5,471**	**$12,513**
Equity Investment	$14,430	$8,683	$2,773	$4,607	$7,813
Bond Investment	$2,652	$12,737	$3,856	$864	$4,700
Mean Equity Return	5.07%	4.93%	11.54%	10.38%	7.95%
Mean Bond Return	3.10%	–1.04%	4.53%	8.53%	3.99%
Mean Equity Premium	**1.97%**	**5.97%**	**7.01%**	**1.85%**	**3.97%**

	***IRA:* α_1 = 4.00 α_2 = 4.25**				
	State 1	**State 2**	**State 3**	**State 4**	**Average**
Middle-Aged Consumption	**$49,759**	**$42,505**	**$30,520**	**$32,188**	**$38,966**
Old Consumption	**$49,441**	**$17,093**	**$68,680**	**$27,410**	**$40,433**
Young Consumption	$19,000	$19,000	$19,000	$19,000	$19,000
Savings/Investment	**$6,091**	**$13,345**	**$2,930**	**$1,262**	**$5,684**
Equity Investment	$4,784	$3,484	$782	$985	$2,546
Bond Investment	$1,307	$9,862	$2,148	$277	$3,138
Mean Equity Return	8.45%	5.82%	13.45%	15.22%	10.85%
Mean Bond Return	3.31%	–0.75%	4.73%	9.84%	4.52%
Mean Equity Premium	**5.14%**	**6.57%**	**8.72%**	**5.38%**	**6.33%**

Table 5. Consumption and Savings/Investment: CRRA vs. IRA Preferences

This table presents consumption levels of the three age cohorts under different model economies. It also shows the pattern of savings of the middle-aged. Φ^S is the share of wealth saved/invested. Results are reported for CRRA and IRA preferences.

	CRRA	IRA				
	$\alpha_1 = 2.00$ $\alpha_2 = 2.00$	$\alpha_1 = 2.00$ $\alpha_2 = 2.05$	$\alpha_1 = 2.00$ $\alpha_2 = 2.10$	$\alpha_1 = 2.00$ $\alpha_2 = 2.15$	$\alpha_1 = 2.00$ $\alpha_2 = 2.20$	$\alpha_1 = 2.00$ $\alpha_2 = 2.25$
Middle-Aged Consumption	$32,288	$35,041	$37,350	$39,205	$40,639	$41,715
Old Consumption	$47,111	$44,358	$42,049	$40,194	$38,760	$37,684
Young Consumption	$19,000	$19,000	$19,000	$19,000	$19,000	$19,000
Savings/Investment	$12,362	$9,609	$7,300	$5,446	$4,011	$2,935
Share of Savings (Φ^S)	**27.7%**	**21.5%**	**16.4%**	**12.2%**	**9.0%**	**6.6%**

	CRRA	IRA				
	$\alpha_1 = 4.00$ $\alpha_2 = 4.00$	$\alpha_1 = 4.00$ $\alpha_2 = 4.05$	$\alpha_1 = 4.00$ $\alpha_2 = 4.10$	$\alpha_1 = 4.00$ $\alpha_2 = 4.15$	$\alpha_1 = 4.00$ $\alpha_2 = 4.20$	$\alpha_1 = 4.00$ $\alpha_2 = 4.25$
Middle-Aged Consumption	$32,137	$33,743	$35,246	$36,629	$37,873	$38,966
Old Consumption	$47,262	$45,656	$44,153	$42,770	$41,526	$40,433
Young Consumption	$19,000	$19,000	$19,000	$19,000	$19,000	$19,000
Savings/Investment	$12,513	$10,907	$9,404	$8,022	$6,777	$5,684
Share of Savings (Φ^S)	**28.0%**	**24.4%**	**21.1%**	**18.0%**	**15.2%**	**12.7%**

	CRRA	IRA				
	$\alpha_1 = 6.00$ $\alpha_2 = 6.00$	$\alpha_1 = 6.00$ $\alpha_2 = 6.05$	$\alpha_1 = 6.00$ $\alpha_2 = 6.10$	$\alpha_1 = 6.00$ $\alpha_2 = 6.15$	$\alpha_1 = 6.00$ $\alpha_2 = 6.20$	$\alpha_1 = 6.00$ $\alpha_2 = 6.25$
Middle-Aged Consumption	$31,590	$32,732	$33,840	$34,908	$35,928	$36,893
Old Consumption	$47,809	$46,667	$45,559	$44,491	$43,471	$42,506
Young Consumption	$19,000	$19,000	$19,000	$19,000	$19,000	$19,000
Savings/Investment	$13,060	$11,918	$10,810	$9,743	$8,722	$7,757
Share of Savings (Φ^S)	**29.2%**	**26.7%**	**24.2%**	**21.8%**	**19.5%**	**17.4%**

Table 6. Portfolio Allocations: CRRA vs. IRA Preferences

This table reports investment in equity and bonds, portfolio shares, and portfolio allocation. Φ^e is the share of wealth invested in equity; Φ^b is the share of wealth invested in bonds; $\boldsymbol{\omega}^e$ is the portfolio share invested in equity; $\boldsymbol{\omega}^b$ is the portfolio share invested in bonds. Results are reported for CRRA and IRA preferences.

	CRRA	IRA				
	$\alpha_1 = 2.00$ $\alpha_2 = 2.00$	$\alpha_1 = 2.00$ $\alpha_2 = 2.05$	$\alpha_1 = 2.00$ $\alpha_2 = 2.10$	$\alpha_1 = 2.00$ $\alpha_2 = 2.15$	$\alpha_1 = 2.00$ $\alpha_2 = 2.20$	$\alpha_1 = 2.00$ $\alpha_2 = 2.25$
Investment in Equity	$9,661	$7,156	$5,104	$3,519	$2,362	$1,557
Investment in Bonds	$2,701	$2,453	$2,196	$1,926	$1,650	$1,378
Equity Share % of Wealth (Φ^e)	21.6%	16.0%	11.4%	7.9%	5.3%	3.5%
Bond Share % of Wealth (Φ^b)	6.0%	5.5%	4.9%	4.3%	3.7%	3.1%
Portfolio Allocation: Equity ($\boldsymbol{\omega}^e$)	**78.2%**	**74.5%**	**69.9%**	**64.6%**	**58.9%**	**53.1%**
Portfolio Allocation: Bonds ($\boldsymbol{\omega}^b$)	**21.9%**	**25.5%**	**30.1%**	**35.4%**	**41.1%**	**47.0%**

	CRRA	IRA				
	$\alpha_1 = 4.00$ $\alpha_2 = 4.00$	$\alpha_1 = 4.00$ $\alpha_2 = 4.05$	$\alpha_1 = 4.00$ $\alpha_2 = 4.10$	$\alpha_1 = 4.00$ $\alpha_2 = 4.15$	$\alpha_1 = 4.00$ $\alpha_2 = 4.20$	$\alpha_1 = 4.00$ $\alpha_2 = 4.25$
Investment in Equity	$7,813	$6,486	$5,287	$4,226	$3,311	$2,546
Investment in Bonds	$4,700	$4,421	$4,117	$3,796	$3,466	$3,138
Equity Share % of Wealth (Φ^e)	17.5%	14.5%	11.8%	9.5%	7.4%	5.7%
Bond Share % of Wealth (Φ^b)	10.5%	9.9%	9.2%	8.5%	7.8%	7.0%
Portfolio Allocation: Equity ($\boldsymbol{\omega}^e$)	**62.4%**	**59.5%**	**56.2%**	**52.7%**	**48.9%**	**44.8%**
Portfolio Allocation: Bonds ($\boldsymbol{\omega}^b$)	**37.6%**	**40.5%**	**43.8%**	**47.3%**	**51.2%**	**55.2%**

	CRRA	IRA				
	$\alpha_1 = 6.00$ $\alpha_2 = 6.00$	$\alpha_1 = 6.00$ $\alpha_2 = 6.05$	$\alpha_1 = 6.00$ $\alpha_2 = 6.10$	$\alpha_1 = 6.00$ $\alpha_2 = 6.15$	$\alpha_1 = 6.00$ $\alpha_2 = 6.20$	$\alpha_1 = 6.00$ $\alpha_2 = 6.25$
Investment in Equity	$7,449	$6,594	$5,777	$5,000	$4,269	$3,589
Investment in Bonds	$5,611	$5,324	$5,033	$4,742	$4,453	$4,168
Equity Share % of Wealth (Φ^e)	16.7%	14.8%	12.9%	11.2%	9.6%	8.0%
Bond Share % of Wealth (Φ^b)	12.6%	11.9%	11.3%	10.6%	10.0%	9.3%
Portfolio Allocation: Equity ($\boldsymbol{\omega}^e$)	**57.0%**	**55.3%**	**53.4%**	**51.3%**	**48.9%**	**46.3%**
Portfolio Allocation: Bonds ($\boldsymbol{\omega}^b$)	**43.0%**	**44.7%**	**46.6%**	**48.7%**	**51.1%**	**53.7%**

Table 7. Absolute and Relative Increase in Risk Aversion

This table reports the impact of risk-averse differential over the life cycle on savings/investment, portfolio shares, and security returns. Φ^S is the share of wealth saved/invested; Φ^e is the share of wealth invested in equity; Φ^b is the share of wealth invested in bonds; ω^e is the portfolio share invested in equity; and ω^b is the portfolio share invested in bonds.

Panel A. Risk Aversion Differential over the Life Cycle

	$\alpha_1 = 2.00$ $\alpha_2 = 2.50$	$\alpha_1 = 6.00$ $\alpha_2 = 6.25$
Mean Equity Return	**14.7%**	**10.83%**
St. Dev of Equity Return	24.6%	24.12%
Mean Bond Return	**6.6%**	**4.43%**
St. Dev of Bond Return	18.7%	20.23%
Mean Equity Premium	**8.2%**	**6.40%**
St. Dev of Equity Premium	24.2%	22.49%
Savings/Investment	**1,872**	**7,757**
Investment in Equity	597	3,589
Investment in Bonds	1,275	4,168
Share of Savings (Φ^S)	**4.19%**	**17.4%**
Equity Share % of Wealth (Φ^e)	1.34%	8.04%
Bond Share % of Wealth (Φ^b)	2.86%	9.33%
Portfolio Allocation: Equity (ω^e)	**31.9%**	**46.3%**
Portfolio Allocation: Bonds (ω^b)	**68.1%**	**53.7%**

Panel B. Constant Absolute Increase in Risk Aversion

	$\alpha_1 = 2.00$ $\alpha_2 = 2.25$	$\alpha_1 = 4.00$ $\alpha_2 = 4.25$	$\alpha_1 = 6.00$ $\alpha_2 = 6.25$
Mean Equity Return	**10.97%**	**10.85%**	**10.78%**
St. Dev of Equity Return	22.71%	23.10%	24.12%
Mean Bond Return	**5.93%**	**4.52%**	**4.38%**
St. Dev of Bond Return	16.64%	18.76%	20.23%
Mean Equity Premium	**5.04%**	**6.33%**	**6.40%**
St. Dev of Equity Premium	22.98%	22.28%	22.49%
Savings/Investment	**$2,935**	**$5,684**	**$7,757**
Investment in Equity	$1,557	$2,546	$3,589
Investment in Bonds	$1,378	$3,138	$4,168
Share of Savings (Φ^S)	6.6%	12.7%	17.4%
Equity Share % of Wealth (Φ^e)	3.5%	5.7%	8.0%
Bond Share % of Wealth (Φ^b)	3.1%	7.0%	9.3%
Portfolio Allocation: Equity (ω^e)	**53.1%**	**44.8%**	**46.3%**
Portfolio Allocation: Bonds (ω^b)	**47.0%**	**55.2%**	**53.7%**

Panel C. Constant Relative Increase in Risk Aversion

	$\alpha_1 = 2.00$ $\alpha_2 = 2.10$	$\alpha_1 = 4.00$ $\alpha_2 = 4.20$	$\alpha_1 = 6.00$ $\alpha_2 = 6.30$
Mean Equity Return	**7.91%**	**10.10%**	**11.63%**
St. Dev of Equity Return	18.43%	22.49%	24.32%
Mean Bond Return	**4.78%**	**4.33%**	**4.76%**
St. Dev of Bond Return	13.89%	18.31%	20.56%
Mean Equity Premium	**3.13%**	**5.77%**	**6.87%**
St. Dev of Equity Premium	19.09%	22.37%	23.40%
Savings/Investment	**7,300**	**6,777**	**6,858**
Investment in Equity	5,104	3,311	2,971
Investment in Bonds	2,196	3,466	3,887
Share of Savings (Φ^S)	**16.4%**	**15.2%**	**15.4%**
Equity Share % of Wealth (Φ^e)	11.4%	7.4%	6.7%
Bond Share % of Wealth (Φ^b)	4.92%	7.76%	8.71%
Portfolio Allocation: Equity (ω^e)	**69.9%**	**48.9%**	**43.32%**
Portfolio Allocation: Bonds (ω^b)	**30.1%**	**51.2%**	**56.68%**

Table 8. The Effect of IRA Preferences under Different Correlation Structures

This table reports results for different correlation structures *a)* $Corr(y_t,w^1_t) = 0.1$; $Corr(y_t,y_{t-1}) = Corr(w^1_t,w^1_{t-1}) = 0.8$; *b)* $Corr(y_t,w^1_t) = 0.8$; $Corr(y_t,y_{t-1}) = Corr(w^1_t,w^1_{t-1}) = 0.1$, *c)* $Corr(y_t,w^1_t) = 0.8$; $Corr(y_t,y_{t-1}) = Corr(w^1_t,w^1_{t-1}) = 0.8$. Φ^S is the share of wealth saved/invested; Φ^e is the share of wealth invested in equity; Φ^b is the share of wealth invested in bonds; $\boldsymbol{\omega^e}$ is the portfolio share invested in equity; and $\boldsymbol{\omega^b}$ is the portfolio share invested in bonds.

	$Corr(y_t,w^1_t) = 0.1$ $Corr(y_t,y_{t-1}) = 0.8$ $Corr(w^1_t,w^1_{t-1}) = 0.8$			$Corr(y_t,w^1_t) = 0.8$ $Corr(y_t,y_{t-1}) = 0.1$ $Corr(w^1_t,w^1_{t-1}) = 0.1$			$Corr(y_t,w^1_t) = 0.8$ $Corr(y_t,y_{t-1}) = 0.8$ $Corr(w^1_t,w^1_{t-1}) = 0.8$		
	$\alpha_1 = 2.00$ $\alpha_2 = 2.25$	$\alpha_1 = 4.00$ $\alpha_2 = 4.25$	$\alpha_1 = 6.00$ $\alpha_2 = 6.25$	$\alpha_1 = 2.00$ $\alpha_2 = 2.25$	$\alpha_1 = 4.00$ $\alpha_2 = 4.25$	$\alpha_1 = 6.00$ $\alpha_2 = 6.25$	$\alpha_1 = 2.00$ $\alpha_2 = 2.25$	$\alpha_1 = 4.00$ $\alpha_2 = 4.25$	$\alpha_1 = 6.00$ $\alpha_2 = 6.25$
Mean Equity Return	**11.57%**	**11.20%**	**11.52%**	**10.62%**	**11.03%**	**12.05%**	**10.22%**	**10.94%**	**11.5%**
St. Dev of Equity Return	23.67%	21.18%	22.03%	16.78%	21.10%	22.30%	16.01%	18.06%	19.21%
Mean Bond Return	**5.98%**	**4.85%**	**4.55%**	**7.20%**	**6.38%**	**6.92%**	**7.28%**	**6.92%**	**6.74%**
St. Dev of Bond Return	15.22%	18.20%	19.44%	16.86%	19.30%	20.10%	13.67%	13.90%	14.61%
Mean Equity Premium	**5.59%**	**6.55%**	**6.97%**	**3.42%**	**4.65%**	**5.13%**	**2.94%**	**4.02%**	**4.84%**
St. Dev of Eq. Premium	20.46%	21.44%	22.17%	16.9%	17.8%	18.0%	14.3%	17.7%	18.6%
Savings/Investment	**2,886**	**5,123**	**6,770**	**2,070**	**3,886**	**6,004**	**1,921**	**2,936**	**4,052**
Investment in Equity	1,401	2,017	2,436	1,265	2,150	3,110	1,222	1,711	2,217
Investment in Bonds	1,485	3,106	4,334	805	1,736	2,894	699	1,224	1,835
Share of Savings (Φ^S)	**6.5%**	**11.5%**	**15.2%**	**4.6%**	**8.7%**	**13.4%**	**4.3%**	**6.6%**	**9.1%**
Equity Share % of Wealth (Φ^e)	3.1%	4.5%	5.5%	2.8%	4.8%	7.0%	2.7%	3.8%	5.0%
Bond Share % of Wealth (Φ^b)	3.3%	7.0%	9.7%	1.8%	3.9%	6.5%	1.6%	2.7%	4.1%
Portfolio Allocation: Equity (ω^e)	**48.6%**	**39.4%**	**36.0%**	**61.1%**	**55.3%**	**51.8%**	**64%**	**58%**	**55%**
Portfolio Allocation: Bonds (ω^b)	**51.5%**	**60.6%**	**64.0%**	**38.9%**	**44.7%**	**48.2%**	**36%**	**42%**	**45%**

Table 9. Scale Effects: Security Returns and Equity Premium with IRA Preferences

The impact of the change in the scale of the economy on security returns and on the equity premium. Results are shown for IRA preferences for model economies that display the same relative increase in risk aversion. All statistics are reported in annualized percentage terms.

	IRA		
	$\alpha_1 = 2.00$ $\alpha_2 = 2.10$ *E(y) = 98,399*	$\alpha_1 = 2.00$ $\alpha_2 = 2.10$ *E(y) = 196,798*	$\alpha_1 = 2.00$ $\alpha_2 = 2.10$ *E(y) = 295,197*
Mean Equity Return	**7.91%**	**8.08%**	**8.19%**
St. Dev of Equity Return	18.43%	18.72%	18.90%
Mean Bond Return	**4.78%**	**4.88%**	**4.95%**
St. Dev of Bond Return	13.89%	14.12%	14.25%
Mean Equity Premium	**3.13%**	**3.20%**	**3.24%**
St. Dev of Equity Premium	19.09%	19.41%	19.59%

	IRA		
	$\alpha_1 = 4.00$ $\alpha_2 = 4.20$ *E(y) = 98,399*	$\alpha_1 = 4.00$ $\alpha_2 = 4.20$ *E(y) = 196,798*	$\alpha_1 = 4.00$ $\alpha_2 = 4.20$ *E(y) = 295,197*
Mean Equity Return	**10.10%**	**10.40%**	**10.58%**
St. Dev of Equity Return	22.49%	22.92%	23.17%
Mean Bond Return	**4.33%**	**4.49%**	**4.59%**
St. Dev of Bond Return	18.31%	18.70%	18.93%
Mean Equity Premium	**5.77%**	**5.91%**	**5.99%**
St. Dev of Equity Premium	22.37%	22.63%	22.78%

	IRA		
	$\alpha_1 = 6.00$ $\alpha_2 = 6.30$ *E(y) = 98,399*	$\alpha_1 = 6.00$ $\alpha_2 = 6.30$ *E(y) = 196,798*	$\alpha_1 = 6.00$ $\alpha_2 = 6.30$ *E(y) = 295,197*
Mean Equity Return	**11.63%**	**13.16%**	**13.40%**
St. Dev of Equity Return	24.32%	31.22%	31.60%
Mean Bond Return	**4.76%**	**6.08%**	**6.20%**
St. Dev of Bond Return	20.56%	27.33%	27.69%
Mean Equity Premium	**6.87%**	**7.08%**	**7.20%**
St. Dev of Equity Premium	23.40%	31.06%	31.13%

Table 10. Growth Effects: Security Returns and Equity Premium with IRA Preferences

The impact of growth on savings/investment, portfolio shares, and security returns. Φ^S is the share of wealth saved/invested; Φ^e is the share of wealth invested in equity; Φ^b is the share of wealth invested in bonds; ω^e is the portfolio share invested in equity; and ω^b is the portfolio share invested in bonds. Results are shown for IRA preferences for model economies that display the same relative increase in risk aversion.

	IRA					
	$\alpha_1 = 2.00$ $\alpha_2 = 2.10$ *n = 0%*	$\alpha_1 = 2.00$ $\alpha_2 = 2.10$ *n = 2%*	$\alpha_1 = 4.00$ $\alpha_2 = 4.20$ *n = 0%*	$\alpha_1 = 4.00$ $\alpha_2 = 4.20$ *n = 2%*	$\alpha_1 = 6.00$ $\alpha_2 = 6.30$ *n = 0%*	$\alpha_1 = 6.00$ $\alpha_2 = 6.30$ *n = 2%*
Mean Equity Return	**7.91%**	**7.92%**	**10.10%**	**10.60%**	**11.63%**	**12.33%**
St. Dev of Equity Return	18.43%	19.19%	22.49%	23.83%	24.32%	25.65%
Mean Bond Return	**4.78%**	**4.49%**	**4.33%**	**3.89%**	**4.76%**	**4.44%**
St. Dev of Bond Return	13.89%	14.28%	18.31%	18.53%	20.56%	20.81%
Mean Equity Premium	**3.13%**	**3.43%**	**5.77%**	**6.71%**	**6.87%**	**7.89%**
St. Dev of Equity Premium	19.09%	21.47%	22.37%	23.93%	23.40%	25.85%
Savings/Investment	**$7,300**	**$7,264**	**$6,777**	**$6,729**	**$6,858**	**$6,800**
Investment in Equity	$5,104	$4,739	$3,311	$2,495	$2,971	$2,007
Investment in Bonds	$2,196	$2,525	$3,466	$4,234	$3,887	$4,793
Share of Savings (Φ^S)	**16.4%**	**16.3%**	**15.2%**	**15.1%**	**15.4%**	**15.2%**
Equity Share % of Wealth (Φ^e)	11.4%	10.6%	7.4%	5.6%	6.7%	4.5%
Bond Share % of Wealth (Φ^b)	4.9%	5.7%	7.8%	9.5%	8.7%	10.7%
Portfolio Allocation: Equity (ω^e)	**69.9%**	**65.2%**	**48.9%**	**37.1%**	**43.3%**	**29.5%**
Portfolio Allocation: Bonds (ω^b)	**30.1%**	**34.8%**	**51.2%**	**62.9%**	**56.7%**	**70.5%**

Table 11. Pension Scheme: Security Returns and Equity Premium with IRA Preferences

The impact of pension income on savings/investment, portfolio shares, and security returns. Φ^S is the share of wealth saved/invested; Φ^e is the share of wealth invested in equity; Φ^b is the share of wealth invested in bonds; ω^e is the portfolio share invested in equity; and ω^b is the portfolio share invested in bonds. Results are reported for CRRA and IRA preferences.

	CRRA				IRA			
	$\alpha_1 = 4.00$ $\alpha_2 = 4.00$ $T = 0\%$	$\alpha_1 = 4.00$ $\alpha_2 = 4.00$ $T = 6.4\%$	$\alpha_1 = 4.00$ $\alpha_2 = 4.00$ $T = 12.4\%$	$\alpha_1 = 4.00$ $\alpha_2 = 4.00$ $T = 15\%$	$\alpha_1 = 4.00$ $\alpha_2 = 4.20$ $T = 0\%$	$\alpha_1 = 4.00$ $\alpha_2 = 4.20$ $T = 6.4\%$	$\alpha_1 = 4.00$ $\alpha_2 = 4.20$ $T = 12.4\%$	$\alpha_1 = 4.00$ $\alpha_2 = 4.20$ $T = 15\%$
Mean Equity Return	**7.95%**	**8.37%**	**9.14%**	**9.52%**	**10.10%**	**10.71%**	**11.85%**	**11.96%**
St. Dev of Equity Return	20.56%	21.71%	23.35%	23.65%	22.49%	22.75%	23.50%	24.21%
Mean Bond Return	**3.99%**	**4.09%**	**4.56%**	**4.81%**	**4.33%**	**4.44%**	**5.26%**	**5.30%**
St. Dev of Bond Return	17.21%	18.27%	19.78%	21.69%	18.31%	18.44%	18.81%	19.25%
Mean Equity Premium	**3.96%**	**4.28%**	**4.58%**	**4.71%**	**5.77%**	**6.27%**	**6.59%**	**6.66%**
St. Dev of Eq. Premium	21.01%	22.72%	23.52%	24.30%	22.37%	23.13%	23.81%	23.98%
Savings/Investment	**$12,513**	**$9,892**	**$7,531**	**$6,619**	**$6,777**	**$4,726**	**$3,085**	**$2,506**
Investment in Equity	$7,813	$6,040	$4,493	$3,912	$3,311	$2,174	$1,355	$1,089
Investment in Bonds	$4,700	$3,852	$3,038	$2,707	$3,466	$2,552	$1,730	$1,417
Share of Savings (Φ^S)	**28.0%**	**22.2%**	**16.9%**	**14.8%**	**15.2%**	**10.6%**	**6.9%**	**5.6%**
Equity Share % of Wealth (Φ^e)	17.5%	13.5%	10.1%	8.8%	7.4%	4.9%	3.0%	2.4%
Bond Share % of Wealth (Φ^b)	10.5%	8.6%	6.8%	6.1%	7.8%	5.7%	3.9%	3.2%
Portfolio Allocation: Equity (ω^e)	**62.4%**	**61.1%**	**59.7%**	**59.1%**	**48.9%**	**46.0%**	**43.9%**	**43.5%**
Portfolio Allocation: Bonds (ω^b)	**37.6%**	**38.9%**	**40.3%**	**40.9%**	**51.2%**	**54.0%**	**56.1%**	**56.5%**