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The demographic transition and
the business asset supply channel

Pedro Amaral

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The demographic transition and the business asset supply channel*

Pedro S. Amaral
California State University, Fullerton

Abstract

This paper examines the macroeconomic consequences of a demographic transition in an environment where households can hold diverse portfolios of risky and safe assets at different points in their life cycle. These assets are issued by businesses in order to finance their activities. Businesses are heterogeneous with respect to how productive they are in different states of the world, and therefore pursue different combinations of safe and/or risky securities issuance when financing projects. I simulate a demographic transition calibrated to replicate the expected demographic evolution of current West African economies. This results in modest increases in output, larger increases in saving as a whole and, in particular, in a relative increase in saving in the form of safe assets. Lower capital costs lead to business entry (and more asset issuance) and to a tilt towards safe asset issuance; a mechanism I call the *business asset supply* channel. I show that omitting this channel, as models with a representative firm do, results in an overestimation of the effects of the demographic transition, with larger interest rate reductions and an exaggerated *demographic dividend*.

Keywords: Demographic Transition, Demographic Dividend, Financing Policy, Overlapping Generations

JEL codes: E21, E43, E44, G32, J11

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1 Introduction

The macroeconomic impact of demographic transitions – the process by which societies move from a context of high birth and death rates to one where both rates are relatively lower – is a subject that has been extensively studied by both demographers and economists. Important transmission channels like the evolution of the labor force, changes in savings and interest rates, implications for average labor productivity, and the impact on ideas, innovation and business startup rates, have been identified, but the debate is as alive as ever, fueled by population aging concerns in developed economies.¹

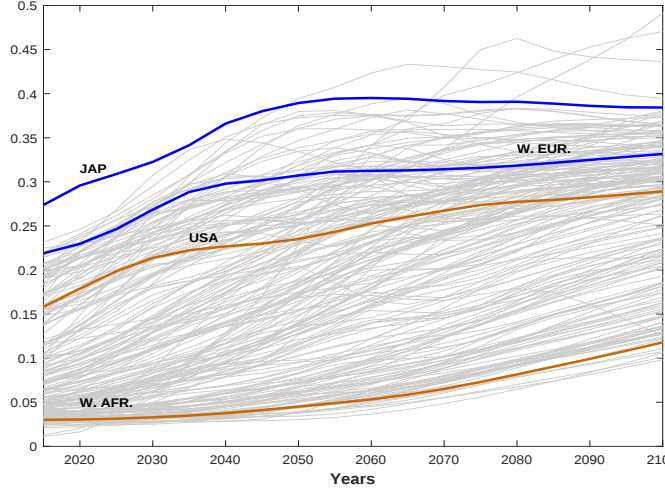
While a lot of the recent research has focused on how population aging will impact advanced economies like Japan, the U.S. or Western European countries, many countries around the world are still in the early stages of their demographic transition. Figure 1 shows the predicted evolution of the share of population older than 65 and conveys just how diverse the current cross-country situation is and will continue to be. The demographic transition simulation I conduct is not supposed to portray the near future of a developed economy; it is instead designed to capture the changes a currently young and developing region like West Africa will undergo, as their demographic characteristics move towards those of today’s advanced economies. These details are important because changes to a population’s demographic structure have important consequences not only for aggregate saving, but also for the allocation of saving across different asset types.

The demographic transition simulation is characterized by an increase in household saving and, in particular, by a relative increase in the demand for safe assets.² This, in turn, lowers interest rates (safe rates, in particular) and the cost of capital, encouraging two effects that have important implications for asset markets and macro variables: (1) business entry increases, and therefore so does asset issuance; (2) businesses change their financing asset mix towards safe issuance. These effects push against the downward pressure on interest rates, a mechanism I call the *business asset supply channel*. I argue this is a quantitatively important channel, in the sense that ignoring it leads

¹For each of the channels mentioned, see Gagnon, Johannsen, and Lopez-Salido (2016), Carvalho, Ferrero, and Nechio (2016), Feyrer (2007), Acemoglu and Restrepo (2017), and Karahan, Pugsley, and Sahin (2019). For surveys see Bloom, Canning, and Sevilla (2001), Birdsall, Kelley, and Sinding (2003), Lee (2016), and Iparraguirre (2020).

²In the context of more developed, aging, economies, Bernanke, Bertaut, Demarco, and Kamin (2011) was the first to talk about a *savings glut*, while Caballero, Farhi, and Gourinchas (2017) emphasized that the supply of safe assets, in particular, have not kept up with increases in demand.

Figure 1: Share of the population older than 65 (UN predictions)



to a substantial overestimation of the transition’s impact on interest rates and growth in capital and output – the so-called *demographic dividend*.³

The standard approach designed to capture the impact of a demographic transition on saving and interest rates focuses on how household savings behavior changes over the life-cycle (because of changes in dependency ratios, income profiles and longevity, among others). Households are usually assumed to save in the form of capital and/or a government bond. The production side is usually modeled by assuming a representative firm that rents all, or part, of household savings and uses it in the form of capital.

Here, I depart from this setup by introducing production-side heterogeneity (in addition to the life-cycle heterogeneity on the household side). In the model economy, producers are endowed with different levels of managerial ability in different aggregate states of the world, and can finance their projects by issuing different types of securities: state-contingent and non-state-contingent. The demand for the two types of securities comes from households at different stages in their life-cycle. Pre-retirement and retired households demand riskless securities, while younger households

³For recent examples, besides the ones already cited, of research emphasizing the impact of the demographic transition on macroeconomic variables through changes in savings and interest rates, see [Lisack, Sajedi, and Thwaites \(2017\)](#), [Auclert, Martenet, and Malmberg \(2019\)](#), and [Liu and Poonpolkul \(2020\)](#). On the empirical side, [Lunsford and West \(2019\)](#) find that changes in demographic variables are one of the leading explanations for the decrease in real U.S. interest rates in the last 30 years.

demand state-contingent securities. In the model this is hardwired into preferences (risk-neutral vs. infinitely risk-averse), but this is simply a modeling device designed to capture the fact that older household's portfolios emphasize safer assets. In a recent study using Norwegian household tax records, [Fagereng, Gottlieb, and Guiso \(2017\)](#) find that, as they approach retirement, households substantially reduce their exposure to risky assets, increasing their exposure to safe ones. Using consecutive cohorts, they are able to sidestep two problems that have hitherto plagued attempts to measure how household portfolios vary with age: cohort effects (previous studies, like the evidence cited in [Guiso, Haliassos, and Jappelli \(2002\)](#), looked mostly at cross-sectional data) and endogenous stock-market participation.

Another way in which the model departs from the literature is in allowing for aggregate uncertainty within an overlapping-generations framework: most models in the literature are populated by either perfectly foresighted households, or feature idiosyncratic uncertainty.⁴ The presence of aggregate uncertainty is necessary when dealing with state-contingent and safe assets simultaneously. This allows for time varying interest rates on the different assets and correspondingly time-varying financing policies for businesses.

Like most models in this literature that emphasize the savings channel, I also find that a demographic transition leads to an increase in aggregate saving. Not only because the share of savers increases, but also because of reductions in the marginal propensity to consume (due to increases in expected longevity and reductions in the number of children). The main difference relative to representative firm models is that as saving increases and puts downward pressure on interest rates, producers react and change their financing policies: by issuing more of those securities that see more downward pressure on interest rates, and importantly, by activating projects that were not profitable at higher interest rates. In a counterfactual exercise, I show that the presence of producer-side heterogeneity and the resulting changes in financing policies lead to a sizable difference in the behavior of interest rates, productive capital, and output along the transition.

World economies have been undergoing demographic transitions since the 18th century.⁵ It did not take long for economists and demographers to start speculating about the effects of such

⁴See [Carvalho, Ferrero, and Nechio \(2016\)](#), [Lisack, Sajedi, and Thwaites \(2017\)](#), [Auclert, Martenet, and Malmberg \(2019\)](#), and [Liu and Poonpolkul \(2020\)](#) for recent examples.

⁵See [Delventhal, Fernández-Villaverde, and Gunner \(2019\)](#) for an excellent and thorough survey of demographic transitions across time and around the world.

demographic changes: Malthus published *An Essay on the Principle of Population* in 1798. More recently, Coale and Hoover (1958) was perhaps the first instance where the savings channel, as the nexus between demographic changes and economic growth, was put forward more formally. The term *demographic dividend* loosely captures the benefits from a potential acceleration in growth an economy may experience following a demographic transition, particularly one characterized by reductions in fertility. My work contributes to this literature by arguing that models that attempt to structurally measure the magnitude of the demographic dividend risk severely overestimating it if they ignore the business asset supply channel.⁶

2 The environment

Time is discrete and infinite and there is aggregate uncertainty about the state of the economy. The aggregate shock $\eta \in \{L, H\}$ can be low (L) or high (H) and follows a first-order Markov process with known transition function $M : \{L, H\} \rightarrow \{L, H\}$. I assume that M is irreducible, hence globally ergodic.

There are two types of finitely-lived agents in the economy: households and producers. The latter are endowed with an idiosyncratic project ability that depends on the aggregate shock in a way to be made clear below, but lack funds to start a project. In order to do so, they sell securities to the households, whom I now cover in more detail.⁷

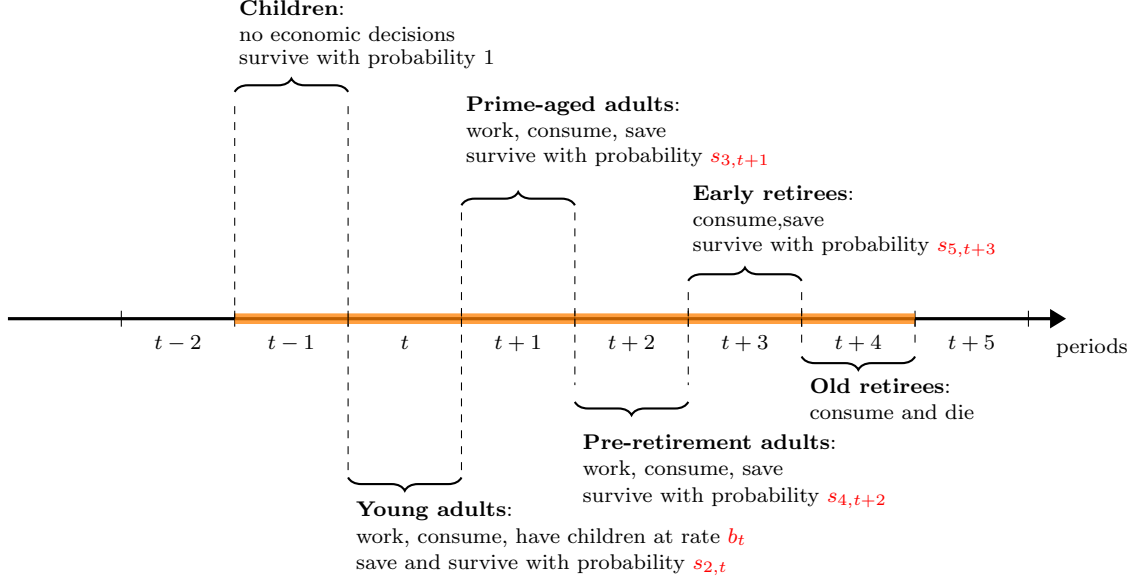
2.1 Households

Each period, a new cohort of households, who may live for a maximum of six periods, is born. The size of each consecutive cohort is time-varying, as birth rates and survival rates may change over time. Given the measure of each cohort $N_{j,t}$, for $j = 1, \dots, 6$, denote the total household population alive at time t by $N_t = \sum_{j=1}^6 N_{j,t}$ and the growth rate of this population by $1 + g_t = N_t/N_{t-1}$. The share of households in each cohort alive at time t is given by $sh_{j,t} = \frac{N_{j,t}}{N_t}$, for $j = 1, \dots, 6$. Each cohort faces a time-varying survival probability given by $s_{j,t}$, for all t and $j = 1, \dots, 6$, with $s_{1,t} = 1$

⁶It is important to emphasize that recent contributions to this literature call into question whether the demographic dividend is brought about by changes in age structure at all, arguing instead that human capital improvements trigger both a demographic transition and the ensuing economic growth (see Galor and Weil (2000)).

⁷A more streamlined, two-period, version of this setup is studied in Amaral and Quintin (2020).

Figure 2: Household timeline



and $s_{6,t} = 0$.

Households value their own lifetime consumption and that of their progeny (while they are children). In broad terms, households start out their lives with a childhood period when they make no meaningful economic decisions: they do not work and their consumption is determined by their parents. For the next three periods, households are endowed with one unit of labor per period which they deliver inelastically in return for a wage that they use for consuming and saving – in the first of these periods, households also have children at a rate, $b_t = \frac{N_{1,t}}{N_{2,t}}$. Finally, there are two retirement periods, when households simply consume the return on their savings. Figure 2 helps envision the households' life cycle.⁸

The households' instantaneous period preferences changes as they age. Households are risk-neutral with respect to consumption bundles all the way up to their pre-retirement period, when they become infinitely risk-averse. As discussed in the Introduction, this is a modeling device designed to capture the kind of life-cycle differences in portfolio composition found in Fagereng, Gottlieb, and Guiso (2017), and in this particular case it implies that households own different types

⁸Note that while I am assuming that age 1 cohorts survive with probability 1, mortality at young ages is still captured in the calibration below through appropriately lower birth rates.

of securities depending on their age. Note that there is no internal inconsistency as far as household decision-making is concerned: households fully anticipate their preferences will change and plan accordingly. I make this assumption because the alternative – having households optimally hold different types of securities simultaneously and having the shares of those holdings change with age – is much more complicated to model, with arguably little upside.

It will help to think of households, in their capacity as investors, as being of two types: one type in their young-adult and prime-aged periods, and another type in their pre-retirement and early retirement periods. Below, in Section 2.2, I assume that producers sell securities to each type separately. I then show that, given household preferences, the securities producers optimally sell to old, infinitely risk-averse, households pay a non-contingent dividend. As a result, I call these securities *safe* (S), while the securities younger agents buy are called *risky* (R), as their dividends may be contingent on the state of the world.

Households observe the period's aggregate shock, η_t , before making any decisions. In the first period of their adult lives they choose a consumption bundle for themselves and their children, along with purchases of securities from producers (to be detailed below), subject to the wage rate they earn. In the second and third periods of their adult lives, they receive not only a wage income, but also the returns on their security portfolios, which they use to consume and update their security portfolio. Throughout their working life, their wage income is subject to a proportional tax τ that funds social security payments. The aggregate tax proceedings are invested for one period at the safe rate of return and then disbursed equally among retirees.⁹ In their first retirement period households may also continue to save, while in their second retirement period they simply consume their income.¹⁰ Finally, there are also incidental bequests: starting in their young adulthood, households receive a bequest in the form of an equal share of the liquidated assets owned by the non-surviving fraction of their parents' cohort.

A household born in period $t - 1$ takes as given wage rates $\{w_{t+j}\}_{j=0}^2$ and the menus of security returns offered by different producers $\{R_{t+j}^i(z, \eta)\}_{j=1}^4$ in each state of the world, for the two different security types $i = R, S$ and chooses a consumption profile $\{c_{j,t+j-2}\}_{j=2}^6$ and security holdings from

⁹The assumption that there is a one-period delay in the pay-as-you go scheme is important for Proposition 1 below.

¹⁰I opted for modeling retirement has having two periods to account for the fact that the increases in life expectancy have not been, in general, accompanied by corresponding increases in retirement age. See, for a similar approach, [Carvalho, Ferrero, and Nechio \(2016\)](#).

each producer z , $\{a_{j,t+j-2}(z)\}_{j=2}^5$ so as to solve:

$$\begin{aligned}
\max E_t & \left\{ \gamma (1 + b_t) \log c_{1,t}(\eta_t) + \log c_{2,t}(\eta_t) + s_{2,t} \beta \log \{E_t(c_{3,t+1}(\eta_{t+1})|\eta_t)\} + s_{3,t+1} \beta^2 \log \{E_{t+1}(c_{4,t+2}(\eta_{t+2})|\eta_{t+1})\} \right. \\
& \quad \left. + s_{4,t+2} \beta^3 \log \{\min(c_{5,t+3}(L), c_{5,t+3}(H))\} + s_{5,t+3} \beta^4 \log \{\min(c_{6,t+4}(L), c_{6,t+4}(H))\} \right\} \\
s.t. \quad & c_{1,t}(\eta_t) + c_{2,t}(\eta_t) + \int_{Z_t} a_{2,t}(z, \eta_t) d\mu \leq (1 - \tau)(1 + \pi_2)w_t + b_{2,t}, \\
& c_{3,t+1}(\eta_{t+1}) + \int_{Z_t} a_{3,t+1}(z, \eta_{t+1}) d\mu \leq (1 - \tau)(1 + \pi_3)w_{t+1} + b_{3,t+1} + \int_{Z_t} a_{2,t}(z, \eta_t) R_{t+1}^R(z) d\mu, \\
& c_{4,t+2}(\eta_{t+2}) + \int_{Z_t} a_{4,t+2}(z, \eta_{t+2}) d\mu \leq (1 - \tau)(1 + \pi_4)w_{t+2} + b_{4,t+2} + \int_{Z_t} a_{3,t+1}(z, \eta_{t+1}) R_{t+2}^R(z) d\mu, \\
& c_{5,t+3}(\eta_{t+3}) + \int_{Z_t} a_{5,t+3}(z, \eta_{t+3}) d\mu \leq b_{5,t+3} + p_{t+3} + \int_{Z_t} a_{4,t+2}(z, \eta_{t+2}) R_{t+3}^S(z) d\mu, \\
& c_{6,t+4}(\eta_{t+4}) \leq p_{t+4} + \int_{Z_t} a_{5,t+3}(z, \eta_{t+3}) R_{t+4}^S(z) d\mu, \\
& c_{1,t}, c_{2,t}, c_{3,t+1}, c_{4,t+2}, c_{5,t+3}, c_{6,t+4} > 0,
\end{aligned}$$

where the incidental bequests are given by

$$\begin{aligned}
b_{2,t} &= \frac{sh_{2,t-1}}{sh_{2,t}} \frac{1 - s_{2,t-1}}{1 + g_t} \int_{Z_t} a_{2,t-1}(z) R_t^R(z) d\mu, \\
b_{3,t+1} &= \frac{sh_{3,t}}{sh_{3,t+1}} \frac{1 - s_{3,t}}{1 + g_{t+1}} \int_{Z_t} a_{3,t}(z) R_{t+1}^R(z) d\mu, \\
b_{4,t+2} &= \frac{sh_{4,t+1}}{sh_{4,t+2}} \frac{1 - s_{4,t+1}}{1 + g_{t+2}} \int_{Z_t} a_{4,t+1}(z, \eta_{t+1}) R_{t+2}^S(z) d\mu, \\
b_{5,t+3} &= \frac{sh_{5,t+2}}{sh_{5,t+3}} \frac{1 - s_{5,t+2}}{1 + g_{t+3}} \int_{Z_t} a_{5,t+2}(z, \eta_{t+2}) R_{t+3}^S(z) d\mu,
\end{aligned}$$

and the social security benefits are given by:

$$\begin{aligned}
p_{t+3} &= \frac{\int_{Z_{t+2}} t_{t+2}(z, \eta_{t+2}) R_{t+3}^S(z) d\mu}{(1 + g_{t+3}) \sum_{j=5}^6 sh_{j,t+3}}, \\
p_{t+4} &= \frac{\int_{Z_{t+3}} t_{t+3}(z, \eta_{t+3}) R_{t+4}^S(z) d\mu}{(1 + g_{t+4}) \sum_{j=5}^6 sh_{j,t+4}},
\end{aligned}$$

where $t_t(z, \eta)$, the amounts invested by the social security fund across producers that issue safe

securities must sum up to the total tax revenues:

$$\begin{aligned}\int_{Z_{t+2}} t_{t+2}(z, \eta_{t+2}) d\mu &= \tau w_{t+2} \sum_{j=2}^4 sh_{j,t+2}(1 + \pi_j), \\ \int_{Z_{t+3}} t_{t+3}(z, \eta_{t+3}) d\mu &= \tau w_{t+3} \sum_{j=2}^4 sh_{j,t+3}(1 + \pi_j).\end{aligned}$$

In making decisions, households must form expectations regarding future earnings, possibly several periods ahead. For example, when young adults decide on how much to save they need to conjecture what their income will be for the remainder of their lifetime. The rational expectations solution to OLG models with various cohorts and aggregate uncertainty is computationally challenging, unlike, say, the perfect foresight solution.¹¹ To make such challenge less burdensome, and because the main computational contribution of this piece is on solving the asset supply problem of heterogeneous producers described below, I posit an alternative expectation formation mechanism. Households form expectations by regressing variables of interest on their information set, which includes the path of all past variables up to the decision moment. In practice, and as shown in Appendix section D, I start by assuming households have perfect foresight over demographic variables and regress future discounted wages and interest rates on lags of the state variables: the aggregate shock and aggregate asset holdings of each security type. In the appendix, I also argue that the resulting estimates share a crucial aspect with their rational expectations counterparts: the fact that agents are very close to being “right” on average.

2.2 Producers

There is a large mass of two-period lived producers, whose size changes at the same rate as the household population. Each endowed with an ability vector $z = (z_L, z_H) \in \mathbb{R}_+^2$, indexing their skills in operating a project in the two states of the world during the first period of their lives. Let $\mu(Z)$ denote the mass of producers living in a given Borel set $\mathcal{Z} \subset \mathbb{R}_+^2$ in each period. In the upcoming calibration, I set this distribution μ in such a way that $z_H > z_L$ for most producers, which means the equilibrium aggregate producer profits and overall production are higher, on average, in good

¹¹See, for example, [Reiter \(2015\)](#).

times (H) compared to bad times (L) – hence the names. But the economy also contains producers whose profits are counter-cyclical, namely those that happen to be very productive in bad times.

In the first period of their lives, producers can choose to operate a project before the aggregate uncertainty shock is realized. In order to do so they must pay an entry cost (proportional to population) of $N_t e \geq 0$ units of the consumption good and commit whatever operational capital $k_t(z) > 0$ they plan to use. A project that is activated and operated by a producer of type z with $k_t \geq 0$ units of capital and $n_t \geq 0$ units of labor yields gross output

$$y(k_t, n_t; z(\eta_t)) = z(\eta_t) (k_t^\alpha n_t^{1-\alpha})^\nu$$

at the end of the period, once state η is realized, and where $\alpha, \nu \in (0, 1)$.

Producers value consumption in both periods of their lives. The consumption bundle of a time t -born producer is a non-negative vector: $(c_{y,t}, c_{o,t+1}(L), c_{o,t+1}(H))$ where $c_{y,t}$ is their consumption at the start of the first period of their life, before the period t shock is realized and production takes place, while $(c_{o,t+1}(L), c_{o,t+1}(H))$ is their second-period consumption, which depends on the realization of the aggregate shock at the end of time t . These consumption profiles are ranked according to linear preferences:

$$c_{y,t} + \epsilon E(c_{o,t+1}(\eta_t) | \eta_{t-1}),$$

where ϵ is a small but positive number. Following the realization of the aggregate shock, conditional on having activated a project with capital k_t , and taking the wage rate w_t as given, a producer of skill z chooses the labor input n_t by solving

$$\Pi(k_t, w_t; z(\eta_t)) \equiv \max_{n_t \geq 0} y(k_t, n_t; z(\eta_t)) - n_t w_t,$$

where Π denotes net operating income.

Since they lack funds to operate and finance the activation of a project, producers who become active obtain external funds by selling securities (claims to their output) to households. The fundamental friction in this process is that selling securities to one type of household (either pre-retirement or younger) is costless, whereas selling securities to two different types carries a cost (proportional to population) $N_t \zeta > 0$. One way to rationalize this is to think of the markets for the two types as

being segmented and subject to different regulations, for example related to retirement, in such a way that operating in multiple security markets is more onerous than operating in a single market. But more broadly, one can think of ζ as proxying for costs associated with managing a more complex capital structure.

I follow [Allen and Gale \(1988\)](#) in assuming that producers take the households' willingness to pay for different securities as given when making their own security issuance decisions. A security is a mapping from the aggregate state to non-negative dividends.¹² Denote by $q_{i,t}(x_{i,t}(L), x_{i,t}(H))$ the price households are willing to pay for a marginal amount of a securities of type $i = R, S$ that has payoffs $(x_{i,t}(L), x_{i,t}(H)) \geq (0, 0)$ at date t in each respective state of the world. Note that so far, I am not requiring safe securities ($i = S$) to have non-contingent payoffs: that will be a consequence of the assumptions on preferences. All I am requiring at this point is that the securities sold to the two different types of households (pre-retirement versus younger) may be different and that selling to both types simultaneously carries a cost.

Producers of type (z_L, z_H) that decide to activate their projects, choose capital $k_t > 0$ and non-negative security payoffs $(x_{i,t}(L), x_{i,t}(H))$ for $i = R, S$, in order to maximize

$$c_{y,t} + \epsilon E(c_{o,t+1}(\eta_t) | \eta_{t-1})$$

subject to

$$c_{y,t} \leq q_{S,t}(x_{S,t}(L), x_{S,t}(H)) + q_{R,t}(x_{R,t}(L), x_{R,t}(H)) - k_t - N_t \left(e + \zeta 1_{\{x_{S,t} > 0, x_{R,t} > 0\}} \right),$$

$$c_{o,t+1}(L) \leq \Pi(k_t, w_t(L); z_L) - x_{S,t}(L) - x_{R,t}(L),$$

$$c_{o,t+1}(H) \leq \Pi(k_t, w_t(H); z_H) - x_{S,t}(H) - x_{R,t}(H),$$

where the indicator function $1_{\{x_{S,t} > 0, x_{R,t} > 0\}}$ takes the value one if both types of securities are issued, in which case the producer must bear cost η , and the value zero otherwise. The first constraint in this problem states that the producer gets to consume whatever is left over of their proceeds from selling securities, $q_{S,t}(x_{S,t}(L), x_{S,t}(H)) + q_{R,t}(x_{R,t}(L), x_{R,t}(H))$, after all costs, $k_t + N_t \left(e + \zeta 1_{\{x_{S,t} > 0, x_{R,t} > 0\}} \right)$, have been covered. Producers only become active if they can meet this constraint, as in that case they can enjoy non-negative consumption compared to their opportunity

¹²Allowing for negative dividends (short-selling) would pose existence problems even in one-period versions of the model. See [Allen and Gale \(1988\)](#).

cost of not becoming active, which is zero. The second and third constraints state that, in each state of the world, producers must be able to cover all their security payments with their operating income.

2.2.1 Security pricing and equilibrium

A key feature of the equilibrium I am about to define is that the households' willingness to pay for securities has to coincide with the pricing functionals implied by the security payments set by producers.

Recall that households take as given the set of securities available at the start of a particular period. From their point of view, they face a menu of security returns:

$$R_t^i(z, \eta) = \frac{x_{i,t}(z, \eta)}{q_{i,t}(x_{i,t}(z, L), x_{i,t}(z, H))},$$

for the securities of type $i = \{S, R\}$ issued by producers of type $z = (z_L, z_H)$, with the convention that $R_t^i(z, \eta) = 0$ if type z is inactive.

Since they have risk-neutral preferences, young-adults and prime-aged adults purchase securities from those producers offering the highest expected return. Therefore, letting

$$\bar{R}_t^R = \max_z M(L|\eta_{t-1}) R_t^R(z, L) + M(H|\eta_{t-1}) R_t^R(z, H),$$

these agents are willing to pay:

$$q_{R,t}(x(L), x(H)) = \frac{M(L|\eta_{t-1}) x(L) + M(H|\eta_{t-1}) x(H)}{\bar{R}_t^R}$$

for a marginal investment in a security with payoff $(x(L), x(H))$ at date t .

Pre-retirement and recently retired agents, on the other hand have Leontieff preferences over their remaining consumption plans. Consider such agents alive at time t and define

$$\bar{R}_{4,t}^S \equiv \frac{\min(c_{5,t}(L), c_{5,t}(H))}{a_{4,t-1}}, \text{ and } \bar{R}_{5,t}^S \equiv \frac{\min(c_{6,t}(L), c_{6,t}(H))}{a_{5,t-1}}$$

as the effective return these agents realize on their investment at the optimal solution to their prob-

lem. Anticipating the definition of equilibrium to simplify exposition, note that because producers sell the same securities to cohorts $j = 4, 5$, we will have $\bar{R}_t^S = \bar{R}_{4,t}^S = \bar{R}_{5,t}^S$.

If $c_{j,t}(H) > c_{j,t}(L)$ at the optimal solution, the willingness to pay for a marginal investment in a security with payoffs $(x(L), x(H))$ of an agent of cohort $j = 4, 5$ is

$$q_{S,t}(x(L), x(H)) = \frac{x(L)}{\bar{R}_t^S}.$$

That is because they only value marginal payoffs in the lowest consumption state in that case. The symmetric condition must hold when $c_{j,t}(H) < c_{j,t}(L)$. Finally, when $c_{5,t}(L) = c_{5,t}(H)$ and $c_{6,t}(L) = c_{6,t}(H)$, which, I will argue below, must hold in equilibrium at all dates,

$$q_{S,t}(x(L), x(H)) = \frac{\min(x(L), x(H))}{\bar{R}_t^S}.$$

Let agents of ages $j = 2, \dots, 5$ enter date 0 with wealth $a_{j,-1} > 0$. Then, the state of the economy at date 0 is fully described by $\Theta_0 = \{a_{j,-1}, \eta_{-1}\}$, $j = 2, \dots, 5$, where $\eta_{-1} \in \{L, H\}$ is the aggregate shock at date $t = -1$. Producers only produce when young hence do not accumulate resources. All active producers must therefore raise all the funds they use from households.

To define an equilibrium it will be helpful to do it in per capita terms, where $\hat{x}_t = \frac{x_t}{N_t}$. A *per capita* equilibrium is then defined, for all dates and for all possible histories of aggregate shocks, as a list of:

- consumption plans $\{\hat{c}_{j,t}(\eta)\}$, for $j = 1, \dots, 6$ and security purchases $\{\hat{a}_{j,t}(\eta, z)\}$, for $j = 2, \dots, 5$ for households;
- a set $Z_t \in \mathcal{Z}$ of active producers and their corresponding consumption plans $(\hat{c}_{y,t}(z), \hat{c}_{o,t+1}(\eta, z))$, capital $\{\hat{k}_t(z)\}$, labor $\{\hat{n}_t(z)\}$, and a menu of security payoffs $\{\hat{x}_{i,t}(z, \eta_t)\}$ for security types $i = S, R$;
- social security purchases of safe securities from each producer z : $\hat{t}_t(z, \eta_t)$;
- a list of prices: wage rates $\{\hat{w}_t(\eta)\}$, payoff pricing functionals $\{\hat{q}_{S,t}, \hat{q}_{R,t}\}$, and the associated returns $\{\bar{R}_t^R(z, \eta_t)\}$ and $\{\bar{R}_t^S(\eta_t)\}$,

such that:

1. Security purchases and consumption plans solve each household's problem;
2. Security menus, capital and labor choices and consumption plans solve each producer's problem;
3. The goods market clears:

$$\begin{aligned} \int_{Z_t} \hat{y}(\hat{k}_t(z)_t, \hat{w}_t(\eta); z) d\mu &= \sum_{j=1}^6 sh_{j,t} \hat{c}_{j,t} + \int_{Z_t} \hat{c}_{y,t}(z) + \hat{c}_{o,t}(z) d\mu \\ &+ \int_{Z_{t+1}} (1 + g_{t+1}) \hat{k}_{t+1}(z) + e + \zeta 1_{\{x(z)_{R,t+1} > 0, x(z)_{S,t+1} > 0\}} d\mu; \end{aligned}$$

4. The market for labor clears:

$$\int_{Z_t} \hat{n}(\hat{w}_t(\eta); z) d\mu = \sum_{j=2}^4 sh_{j,t};$$

5. The market for each security type clears, i.e., for μ -almost each producer type z :

$$\begin{aligned} sh_{2,t} \hat{a}_{2,t}(z) + sh_{3,t} \hat{a}_{3,t}(z) &= \hat{q}_{R,t}(\hat{x}_{R,t}(z, L), \hat{x}_{R,t}(z, H)), \text{ and} \\ \hat{t}_t(z) + sh_{4,t} \hat{a}_{4,t}(z) + sh_{5,t} \hat{a}_{5,t}(z) &= \hat{q}_{S,t}(\hat{x}_{S,t}(z, L), \hat{x}_{S,t}(z, H)); \end{aligned}$$

6. Social security purchases of safe assets equal tax revenues:

$$\int_{Z_t} \hat{t}_t(z) d\mu = \tau \hat{w}_t \sum_{j=2}^4 sh_{j,t} (1 + \pi_j);$$

7. Pricing functionals are consistent with the household's willingness to pay for marginal payoffs, i.e.:

- (a) $\hat{q}_{R,t}(\hat{x}(L), \hat{x}(H)) = \frac{M(L|\eta_{t-1})\hat{x}(L) + M(H|\eta_{t-1})\hat{x}(H)}{R_t^R},$
- (b) $\hat{q}_{S,t}(\hat{x}(L), \hat{x}(H)) = \frac{\min(\hat{x}(L), \hat{x}(H))}{R_t^S}$ if $\hat{c}_{j,t}(L) = \hat{c}_{j,t}(H)$ for both $j = 5, 6,$
- (c) $\hat{q}_{S,t}(\hat{x}(L), \hat{x}(H)) = \frac{\hat{x}(H)}{R_t^S}$ if $\hat{c}_{j,t}(L) < \hat{c}_{j,t}(H)$ for both $j = 5, 6,$
- (d) $\hat{q}_{S,t}(\hat{x}(L), \hat{x}(H)) = \frac{\hat{x}(L)}{R_t^S}$ if $\hat{c}_{j,t}(L) > \hat{c}_{j,t}(H)$ for both $j = 5, 6,$

for all possible securities $(\hat{x}(L), \hat{x}(H)) \geq (0, 0)$ where:

$$\bar{R}_t^R = \max_z M(L|\eta_{t-1}) R_t^R(z, L) + M(H|\eta_{t-1}) R_t^R(z, H),$$

while

$$\bar{R}_t^S = \frac{\min(\hat{c}_{j,t}(L), \hat{c}_{j,t}(H))}{\hat{a}_{j,t-1}} \quad \text{for both } j = 5, 6.$$

The properties of a two-period version of this type of equilibrium are studied in [Amaral and Quintin \(2020\)](#). Some of the results carry through to the present environment. In particular, the following proposition establishes that in equilibrium, retired household consumption is non-state contingent, and therefore, pre-retirement households only buy safe securities. It also establishes that the equilibrium return on risky securities has to be larger than that of safe securities, implying that earlier in life, households choose to purchase risky securities.

Proposition 1. *In any equilibrium, the consumption of retired agents is risk-free and they only purchase safe securities. Furthermore, in any equilibrium, $\bar{R}_t^R \geq \bar{R}_t^S$, with a strict inequality whenever $\zeta > 0$ and a positive mass of producers issue two securities.*

The intuition for the result is straightforward. If the retired agents' consumption was such that $\hat{c}_5(H) > \hat{c}_5(L)$ then, a period before, these agents would pay nothing for the H state payoff on any security, as their marginal valuation of consumption in that state would be zero. Nonetheless, in order for $\hat{c}_5(H) > \hat{c}_5(L)$ to hold, a positive mass of securities with higher payoffs in the H state than in the L state must be sold to these agents a period before. But the producers selling those securities would be strictly better-off either selling the H state payoff to younger agents, or simply consuming it themselves. The case in which $\hat{c}_5(H) < \hat{c}_5(L)$ can be similarly ruled out.¹³

The second part of the property follows by contradiction: if it were the case that $\bar{R}_t^R < \bar{R}_t^S$, then it would not be profitable for any producer to issue securities to pre-retirement agents as they would always be better off selling risk-free assets to younger (risk-neutral) investors and consuming excess profits, but this contradicts market clearing for safe securities.

¹³This argument relies on having contemporaneous consumption depend only on the contemporaneous shock through the rate of return. Indeed, if pension benefits were purely pay-as-you-go, a transfer that depended on today's wages (instead of yesterday's, as it is the case), then the rate of return that would fully stabilize consumption across states would no longer be non-state contingent necessarily.

Another important equilibrium property (again, see [Amaral and Quintin \(2020\)](#) for a formal argument) is that producers who choose to issue safe securities, issue as much of it as possible. That is, their non-contingent dividend payment equals their minimum profit across states. What do they do with the remainder of their profits in the state when their profit is highest? If ζ is low enough they can pay the security creation cost and issue risky securities. If this is too expensive, they simply consume the remainder in the second period of their lives.¹⁴

In conclusion, at any date t , besides producers that are inactive, there can be three types of active producers as far as their financing goes. Some of them only sell safe securities to older, infinitely risk-averse, cohorts: this makes sense for producers whose productivities across states are sufficiently similar. Others only sell risky securities to younger cohorts in the risk-neutral periods of their lives: these are producers whose productivities are sufficiently different across states and, importantly, their productivity in one of the states is so low that it does not make financial sense to pay the security creation fee to sell both types of securities. Finally, somewhere in between, there are producers that sell both types of securities and pay the security creation cost.

Having established the main theoretical properties of the model, we can now turn to the main exercise.

3 Simulating a demographic transition

The main experiment simulates a demographic transition from a steady-state that exhibits the age structure characteristics of today's West Africa, to a steady-state where age characteristics are similar to today's United States. I picked West Africa because it is the region that features the youngest population in the United Nations' *World Population Prospects*. I picked the U.S because there is an abundance of data I can use to calibrate the model economy, and while its population is not the oldest (that being Western Europe or Japan), it is much further along its demographic transition compared to West Africa. The first step is to map the population data into model parameters for the initial steady-state, the final steady-state, and the interim transition. Given data on the number of people in each cohort, $P_{j,t}$, for $j = 1, \dots, 6$, the (model) parameters of interest are cohort survival

¹⁴Note that ϵ is a very small but strictly positive number, meaning that producers who only sell securities to infinitely risk-averse cohorts are better off consuming any excess profits in the second period of their lives, rather than selling the right to those cash-flows in the form of contingent securities to infinitely risk-averse cohorts at zero price.

probabilities $s_{j,t} = \frac{P_{j+1,t+1}}{P_{j,t}}$, where it is assumed that $s_{6,t} = 0$, and birth rates $b_t = \frac{P_{1,t}}{P_{2,t}}$.

Steady-states (or balanced growth paths) are useful model concepts, but are usually not a feature of (real) demographic data: population shares of different cohorts are continuously changing as birth rates and death rates endogenously move. To get around this difficulty, I build *synthetic* steady-states, which I use not only at the start and end of the demographic transition, but also to calibrate the model economy. Let me postpone the calibration discussion to the next section and first describe how I turn the raw population data from West Africa and the U.S. into the initial and final steady-states that bookend the transition.

I start by assuming that each of the 6 model economy cohorts comprises 16 years (henceforth a *model period*) and map the age cohorts from the UN data (cohorts of 5 years) into model economy cohorts: 0-15, 16-31, 32-47, 48-63, 64-79, and 80+. This procedure uses not only current data, but also estimated data for the future evolution of the populations in the two regions under the assumption of no migration – the model is not built to deal with migratory flows. Appendix A provides a more detailed description of this process whose result is summarized in Table 1 containing cohort populations for the two regions in 2020 and 2036.

Table 1: Model-compatible population (thousands)

<u>West Africa</u>						
Years/Cohorts	0-15	16-31	32-47	48-63	64-79	80+
2020	182,527	110,789	63,979	32,082	11,492	993
2036	245,295	173,211	102,820	55,514	19,884	1,775
<u>United States</u>						
Years/Cohorts	0-15	16-31	32-47	48-63	64-79	80+
2020	65,007	72,517	67,569	66,968	45,795	13,147
2036	61,782	64,578	70,841	64,116	55,633	24,305

Next, I compute some statistics associated with the 2020-2036 evolution of these populations: namely the growth rate of the first cohort $g_{t+1}^1 = \frac{P(1,t+1)}{P(1,t)} - 1$, and the survival probabilities of each consecutive cohort $s_{j,t} = \frac{P(j+1,t+1)}{P(j,t)}$, for $j = 1, \dots, 5$.

Using these parameters, I construct the two synthetic steady-states by simulating the evolution

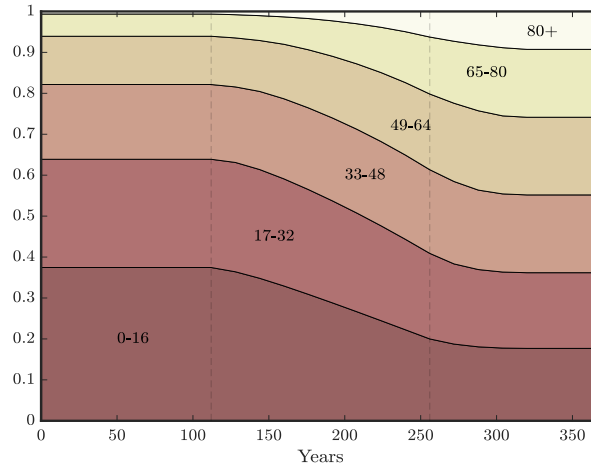
Table 2: Model population parameters

	sh_1	sh_2	sh_3	sh_4	sh_5	sh_6
West Africa: initial steady-state	0.3745	0.2644	0.1826	0.1179	0.0544	0.0062
US: final steady-state	0.1769	0.1849	0.1900	0.1897	0.1658	0.0926
	b	s_1	s_2	s_3	s_4	s_5
West Africa: initial steady-state	0.4162	0.9490	0.9281	0.8677	0.6198	0.1545
US: final steady-state	-0.0433	0.9934	0.9769	0.9489	0.8307	0.5307

of the cohort shares starting from an arbitrary interior point.¹⁵ Table 2 summarizes the resulting steady-state model-relevant moments that are then used as parameters in the model economy.

The *demographic transition* is the path between the two steady-states, which I assume lasts 128 years (8 model periods), with cohort-shares (along with birth rates) adjusting linearly between steady-states as shown in Figure 3. Delventhal, Fernández-Villaverde, and Gunner (2019) report transition lengths between 180 and 50 years. While the evolution of the demographic parameters changes with time, the remaining model parameters are time-invariant and I now go into more details on how they are set.

Figure 3: Cohort shares in transition



¹⁵Convergence to a steady-state in cohort shares occurs very quickly as can be seen in Figure 9 in the appendix for the West Africa case.

3.1 Calibration

To calibrate the model economy to relevant features of the U.S. economy, I start by using the demographic parameter values characterizing a synthetic steady-state resembling the current U.S. demographic characteristics. These are the values shown in Table 2.

Some non-demographic parameters are set exogenously. Given a period length of 16 years, a low state is a rare, but necessarily protracted, event: a disaster in the sense of Barro and Ursua (2008), who define it as a drop in output, from peak to trough, of 10% or more. They find economies spend, on average, 12% of time in those depressed states, but their dataset does not include disasters longer than 16 years. To match this, I set the elements of the aggregate shock's transition matrix M so that the model economy almost always spends at most one period in the low state, $M_{LL} = 0.001$, and the probability of remaining in the high state is set to $M_{HH} = 0.82$, so that the model economy spends 12% of time in the low state.

I set the support of managerial talent to $\mathcal{Z} = [0, 1] \times [0, 1]$, and assume that μ is distributed according to a truncated bivariate log-normal with mean $\bar{z} = (\bar{z}_L, \bar{z}_H)$ and variance-covariance matrix

$$\Phi = \begin{bmatrix} (\varsigma \bar{z}_L)^2 & \sigma \\ \sigma & (\varsigma \bar{z}_H)^2 \end{bmatrix}$$

where $\varsigma > 0$. That is, I take the two skill levels to be correlated at the population level (controlled by σ) and normalize the two variance terms so that the coefficient of variation of managerial talent is the same in the two aggregate states. I set the mean producer productivity in the good state to $\bar{z}_H = 0.05$, a simple normalization.

The income tax rate is set to $\tau = 0.124$, replicating the U.S. social security tax (6.2% on the employee and 6.2% on the employer), but assuming full incidence on the worker. The production function coefficients are ν , which regulates the income share of producer rents, and α , which determines the share of the remaining income accruing to capital. I set the latter to $\alpha = 0.4$ following a vast literature that puts the capital income share in the 35% to 45% range, and calibrate the former below. I set $\epsilon = 10^{-6}$, a number small enough so that ties for producers between consuming left-over output and selling it for nothing are broken in favor of the first option. Finally, to discipline the human capital accumulation, I normalize $\pi_2 = 0$ and set $\pi_3 = 0.28$ and $\pi_4 = 0.16$ so that the

earnings gains in the model match those implied by the findings of [Rupert and Zanella \(2015\)](#) and [Lagakos, Moll, Porzio, Qian, and Schoellman \(2018\)](#). A detailed description of this process can be found in [Appendix B](#).

The remaining parameters are set jointly so as to match selected moments of the steady-state model economy to their U.S. data counterparts, summarized in [Table 3](#).

Table 3: Calibration

Parameters	Targets
$\beta = 0.74$	Risk-free rate: 2.25%
$\sigma = 0.66$	The risk spread is 4.5%
$\bar{z}_L = 0.043$	Drop in output: 17%
$\nu = 0.82$	Producer rents to output: 10%
$\varsigma = 17$	Employment share in 50% smallest projects: 5%
$\gamma = 0.75$	Share of child spending on parental income 32.3%
$e = 0.17$	Entry costs 0.75% of GDP
$c = 0.07$	Security creation costs 0.18% of GDP

I set the discount factor, β , (together with the other parameters) so that the risk-free rate is approximately 2.25% in yearly terms. This is the average (from 1997 to 2019) of the real prime corporate bond yield as measured by the ICE BofA AAA US Corporate Index Effective Yield minus inflation expectations from the Survey of Professional Forecasters.

I set the off-diagonal coefficient of the variance-covariance matrix for skill distribution, σ , such as to obtain an annual risk spread of 4.5%, the average yield spread between the aforementioned AAA yield and the ICE BofA Single-B US High Yield Index Effective Yield, my proxy for risky securities.

The mean managerial talent in the low state, $\bar{z}_L$, is set so that the fall in output (peak to trough) when a bad state occurs is 17%, which is the value I obtain from detrending U.S. output in the [Barro and Ursua \(2008\)](#) dataset using an exponential trend.

The production function parameter regulating managerial profits, ν , is set so that the ratio of producer rents to output in the model is 10%, matching the approximation for this moment obtained by [Corbae and Quintin \(2016\)](#) using US private corporate sector data.

The cross-sectional variance of managerial talent depends on ς , which I set so that the model economy exhibits a share of employment in the 50% smallest projects of about 5%, as in the U.S.

establishment data collected by the Census Bureau in its 2017 County Business Patterns Survey.

How much parents care for their progeny depends on parameter γ , which I set so that in the model economy parents spend the same fraction of their wage income net of taxes as the average U.S. household. In 2015, the USDA estimated that a middle-class married couple spent 16% of pre-tax income per child.¹⁶ A model household with one child is equivalent to a US household with two children. Allowing for a modest degree of economies of scale in child rearing, I assume that raising two children costs 30% of pre-tax income. Finally, the OECD estimates that the tax wedge in the U.S. for an average married worker with two children was at 18.8% in 2019.¹⁷ This implies that γ is set such that in the model, child consumption to second cohort income ratio is $c_1/w = \frac{(1-\tau)}{0.188} 0.3 = 0.323$.

There are two kinds of costs businesses face in the model economy: entry costs, e , and security issuing costs, c . Regarding the former, the World Bank’s *Doing Business project* estimates that the cost of business start-up procedures as a fraction of GNI per capita in the U.S. in 2018 was 1%. On the other hand, [Djankov, Porta, de Silanes, and Shleifer \(2002\)](#) estimate these costs to be roughly half of that, at 0.5% of GDP. Splitting the difference, I set e such that entry costs to GDP in the model are 0.75% of output.

Active projects also need to pay a cost, c , if they choose to issue both types of securities simultaneously. Underwriting fees for corporate debt in the U.S. average roughly 88 basis points (see [Manconi, Neretina, and Renneboog \(2018\)](#)). At the same time, corporate debt issuance in the U.S. has averaged roughly \$2 trillion from 2016 to 2019, according to Moody’s, implying that underwriting fees represented roughly 0.09% of GDP.¹⁸ I take a conservative view and assume other implicit security creation costs – whether in terms of governance, disclosure or managing a more complex capital structure – double these costs to 0.18% of GDP.

¹⁶See “Expenditures on children by families” (USDA, 2015). A middle-class couple is defined as having earned between \$59,200 and \$107,400 in 2015.

¹⁷Note that this differs from the model tax rate $\tau = 0.124$ since in the model, tax revenues are used exclusively for social security funding.

¹⁸See Moody’s Analytics *Weekly Market Report*, November 14, 2019.

3.2 Results

Given a set of parameters and a large enough sequence of aggregate shocks, η_t , the model economy converges to a stochastic steady-state characterized by an invariant distribution.¹⁹ The experiment involves a large number of simulations, $s = 1, \dots, S$, each one characterized by a different sequence of aggregate shocks for $t = 1, \dots, T$ periods: $\{\eta_t^s\}$ drawn from the Markov transition matrix M . Each of these simulations runs for $T = 48$ model periods. The results shown and cited below correspond to the averages over a large number of simulations, and as such, approach the mean of each variable's invariant distribution, given M is irreducible.

The economy starts out in the original steady-state, characterized by the West-African demographic parameters, b and s_j , shown in Table 2. After spending 16 periods there, a transition ensues where b_t and $s_{j,t}$ linearly change over 8 model periods (128 years) until they match the values corresponding to the US demographic characteristics (also shown in the same Table), where it spends another 24 periods.

As Figure 3 makes clear, the demographic transition results in a large increase in the share of retirees – from 6 to 26 percent – entirely at the expense of the share of children in the population. Since early retirees continue to save, this means, first, that the share of the population saving (in any form) increases from 62 percent to 73 percent, and secondly, that most of this increase skews towards safe assets: the share of savers that save in the form of riskless assets jumps from 28 to 49 percent.

The transition results in an increase of roughly 36 percent in GDP per capita, as shown in panel A of figure 4.²⁰ Given that each model period corresponds to 16 years, this translates to a yearly increase of 0.24 percent in GDP per capita. Having noted this, I want to de-emphasize the *absolute* magnitudes because this is, for all its features, a very stylized model. The model does not capture the demographic transition's effect on entrepreneurship, for example, something that can have a deleterious effect on growth (see Liang, Wang, and Lazear (2018) and Aksoy, Basso, Smith, and

¹⁹See, for a standard argument, Brock and Mirman (1972).

²⁰To be clear, whether GDP increases or decreases following a demographic transition in a model like this depends on the nature of the transition. A transition from West-African-like demographic characteristics to US ones moves people to cohorts that are more productive and save more and therefore increases output. In contrast, a transition from US-like demographic characteristics to Japanese-like ones would result in a decline in output because of the relatively larger burden of non-saving retirees.

Grasl (2019)); it also does not allow for the adoption of labor-substituting automation technologies that allow economies to reverse a potential labor scarcity trend, as in Acemoglu and Restrepo (2017).

I do want to emphasize the magnitudes *relative* to a version of the model without the asset supply channel – which I do in the next section. For now, it will be instructive to understand in more detail where this increase is coming from: it is the net result of four proximate channels.

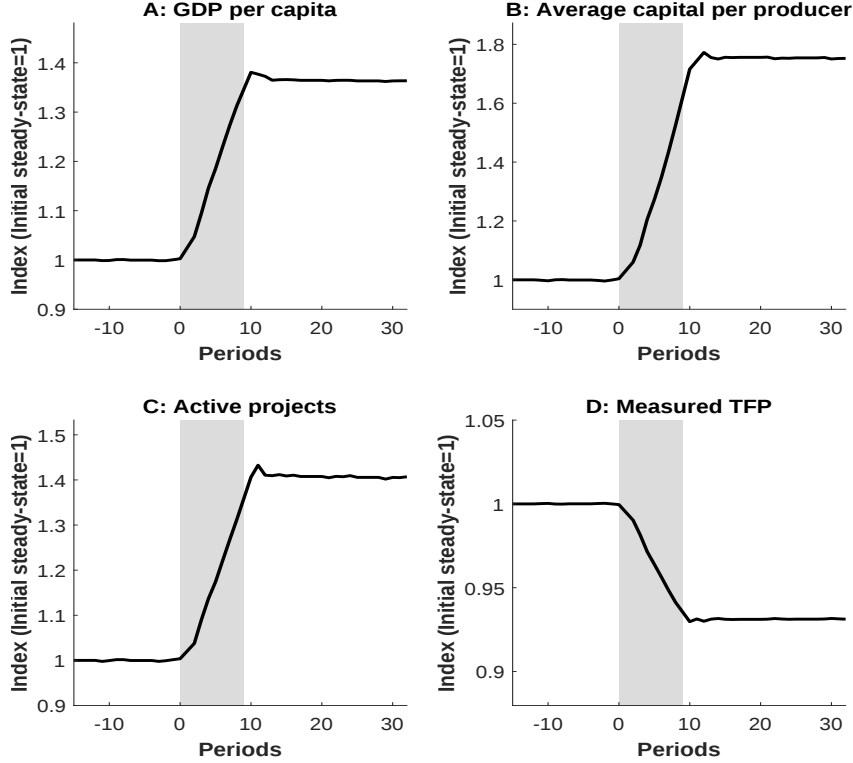
The first three channels are direct consequences of the fact that overall saving in the economy increases. This results, first, in an increase in the average capital each project uses, which can be gauged in panel B of Figure 4. Second, it also gives rise to an increase in the share of active projects by about 40 percent, as shown in panel C of the same figure. The third channel operates against the net increase in income: the newly-activated projects were infra-marginal in the original steady-state, that is, they are operated by less skilled managers and, as such, lower overall measured TFP, as seen in Panel D of Figure 4.

The fourth and final channel is a rather mechanical one, and of less interest for my analysis: the average worker becomes more productive. Even though the share of working-age population remains largely unchanged from one steady-state to another at 46 percent, average worker productivity increases as more workers move to more productive age cohorts. Given the skill evolution calibration (see details in Appendix section B), this shift results in 2.1% higher worker productivity from one steady-state to another. It is therefore clear that this is not the main channel at work as, holding all other factors fixed, this would result in an output increase of $1.021^{0.6 \times 0.82} - 1 = 0.0103$, or about 3 percent of the total increase in output. In Appendix section B.1 I confirm, in a more formal way, the relative little importance of skill evolution for the increase in output by simulating the demographic transition assuming no skill change, that is with $\pi_2 = \pi_3 = \pi_4 = 0$.

The increase in aggregate saving and overall project financing is associated with a reduction in the cost of capital: both the safe and risky rates fall markedly, as shown in panel A of Figure 5. But this is where the similarities end regarding the two security types. As panel B shows, saving in the form of risky securities falls, in contrast to its safe counterpart which increases substantially, more than compensating for the reduction in the former, and increasing overall saving.

To understand what is behind this difference it helps to look at panel D, that plots saving by cohort type: the result of individual household-level saving multiplied by the respective cohort share.

Figure 4: **Aggregate outcomes**

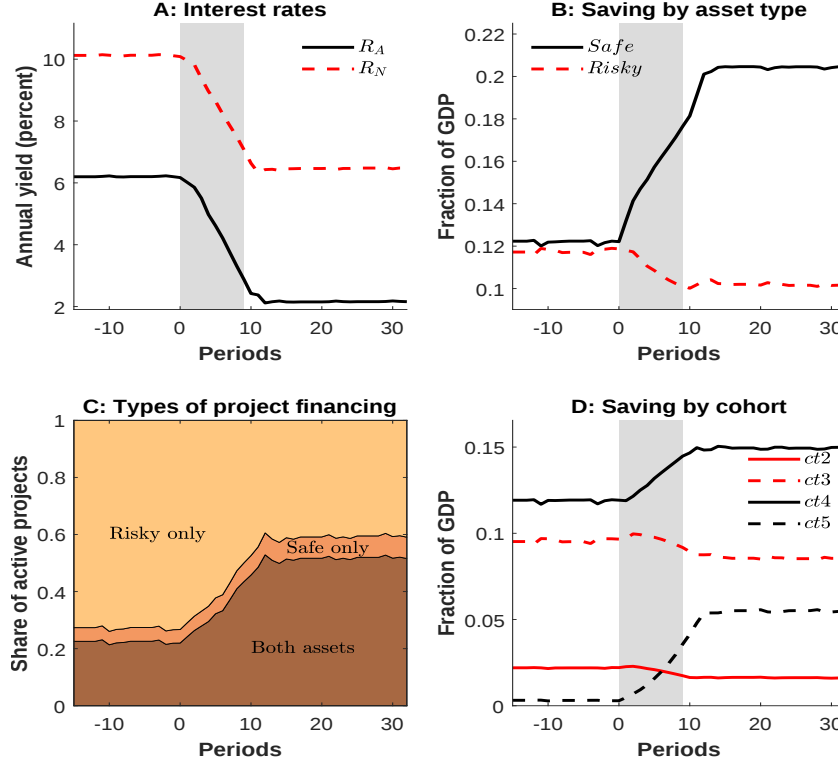


All cohorts increase their individual-level saving. This is because marginal propensities to save out of current income increase at the same time that incomes increase.²¹ That, plus the fact that the shares of cohorts that save in the form of riskless assets (cohorts 4 and 5) increase in the transition, explains the rise in riskless saving. Cohort 2's size, on the other hand, decreases along the transition (see Figure 3), and it does so proportionately more than the increase in individual-level saving, which explains why its overall cohort saving falls in panel D. Finally, while the saving of cohort 3 rises in absolute value, this increase is smaller than that of GDP, and so the ratio of saving to GDP, which is what is plotted in panel D, decreases.

But this is not the end of the story surrounding asset markets; and this is where modeling the asset supply side becomes important. As the increased saving puts downward pressure on interest

²¹Marginal propensities to save increase because life expectancy increases following the demographic transition (see [Carvalho, Ferrero, and Nechio \(2016\)](#) for a similar effect). Importantly, marginal propensities to save out of future income drop (for the same reason) as expected future incomes increase, but this effect is smaller because of discounting.

Figure 5: Saving and asset markets



rates, hitherto idle projects find that capital costs become low enough to justify entry. Entering projects need financing, leading to an increase in asset issuance. Moreover, as the increased demand for safe securities puts relatively more downward pressure on the safe rate (vis-à-vis the risky rate), producers shift financing sources more towards safe securities issuance. In particular, a large mass of producers starts to issue safe securities *in addition to* risky ones (see panel C of Figure 5), securitizing their risky cash-flows, but not so much so as to outstrip the increased demand for safe assets and drive interest rates up. In contrast, risky rates go down even though the demand for risky securities falls. This is the result of the fact that risky asset issuance contracts even more, as more projects substitute away from risky into safe issuance. Absent securitization costs, safe issuance would be preferred by producers since households are willing to pay more for it. With strictly positive securitization costs, as the demand for safe assets increases, the risk premium widens (in this case by 50 basis points). As a result, some of the producers that were issuing risky assets in exclusive realize enough capital cost savings by starting to issue safe assets (in addition to risky

ones) to cover the additional securitization cost.

One of the main findings of this paper is that this safe asset issuance push-back (that characterizes a model with an asset supply channel) dampens the fall in interest rates in a quantitatively important way. Therefore, models that omit this channel risk overestimating the impact of the demographic transition. I show this formally in the next section.

3.3 A representative firm, one asset, counterfactual

As established in the previous section, projects react to the transition-induced saving glut by adjusting their financing policies and/or entering decisions. Since this endogenous reaction gives rise to an increase in security issuance, counterfactually, if financing policies and entry decisions were fixed, interest rates would fall even more, and the impact of the transition on capital and output would be larger. Since this is the central mechanism this article highlights, it is important to ascertain its magnitude. Moreover, if it is the case that this channel turns out to be important, then this calls into question the quantitative predictions of representative firm models of the demographic transition.

To investigate this, I simulate the same demographic transition, but under the assumption that there is a representative firm that requires financing and issues risky debt only. To be clear, although this is the more standard way of modeling the production side, I refer to the model developed in the previous section (with heterogeneous producers) as the *benchmark*.

I follow the same setup and equilibrium concept and, in the interest of brevity, I only show the household and producer problems, highlighting the differences. I relax the assumption that older cohorts are infinitely risk-averse and keep the same risk-neutral period log preferences throughout the household's lifetime. Formally, households now solve:

$$\max E_t \left\{ \gamma (1 + b_t) \log c_{1,t}(\eta_t) + \log c_{2,t}(\eta_t) + s_{2,t} \beta \log \{E_t (c_{3,t+1}(\eta_{t+1}) | \eta_t)\} + s_{3,t+1} \beta^2 \log \{E_t (c_{4,t+2}(\eta_{t+2}) | \eta_{t+1})\} \right. \\ \left. + s_{4,t+2} \beta^3 \log \{E_t (c_{5,t+3}(\eta_{t+3}) | \eta_{t+2})\} + s_{5,t+3} \beta^4 \log \{E_t (c_{6,t+4}(\eta_{t+4}) | \eta_{t+3})\} \right\}$$

$$\begin{aligned}
s.t. \quad c_{1,t}(\eta_t) + c_{2,t}(\eta_t) + \int_{Z_t} a_{2,t}(z, \eta_t) d\mu &\leq (1 - \tau)(1 + \pi_2)w_t + b_{2,t}, \\
c_{3,t+1}(\eta_{t+1}) + \int_{Z_t} a_{3,t+1}(z, \eta_{t+1}) d\mu &\leq (1 - \tau)(1 + \pi_3)w_{t+1} + b_{3,t+1} + \int_{Z_t} a_{2,t}(z, \eta_t) R_{t+1}(z) d\mu, \\
c_{4,t+2}(\eta_{t+2}) + \int_{Z_t} a_{4,t+2}(z, \eta_{t+2}) d\mu &\leq (1 - \tau)(1 + \pi_4)w_{t+2} + b_{4,t+2} + \int_{Z_t} a_{3,t+1}(z, \eta_{t+1}) R_{t+2}(z) d\mu, \\
c_{5,t+3}(\eta_{t+3}) + \int_{Z_t} a_{5,t+3}(z, \eta_{t+3}) d\mu &\leq p_{t+3} + b_{5,t+3} + \int_{Z_t} a_{4,t+2}(z, \eta_{t+2}) R_{t+3}(z) d\mu, \\
c_{6,t+4}(\eta_{t+4}) &\leq p_{t+4} + \int_{Z_t} a_{5,t+3}(z, \eta_{t+2}) R_{t+4}(z) d\mu, \\
c_{1,t}, c_{2,t}, c_{3,t+1}, c_{4,t+2}, c_{5,t+3}, c_{6,t+4} &> 0,
\end{aligned}$$

where the incidental bequests are given by

$$\begin{aligned}
b_{2,t} &= \frac{sh_{2,t-1}}{sh_{2,t}} \frac{1 - s_{2,t-1}}{1 + g_t} \int_{Z_t} a_{2,t-1}(z) R_t(z) d\mu, \\
b_{3,t+1} &= \frac{sh_{3,t}}{sh_{3,t+1}} \frac{1 - s_{3,t}}{1 + g_{t+1}} \int_{Z_t} a_{3,t}(z) R_{t+1}(z) d\mu, \\
b_{4,t+2} &= \frac{sh_{4,t+1}}{sh_{4,t+2}} \frac{1 - s_{4,t+1}}{1 + g_{t+2}} \int_{Z_t} a_{4,t+1}(z, \eta_{t+1}) R_{t+2}(z) d\mu, \\
b_{5,t+3} &= \frac{sh_{5,t+2}}{sh_{5,t+3}} \frac{1 - s_{5,t+2}}{1 + g_{t+3}} \int_{Z_t} a_{5,t+2}(z, \eta_{t+2}) R_{t+3}(z) d\mu,
\end{aligned}$$

and the social security benefits are given by:

$$\begin{aligned}
p_{t+3} &= \frac{\int_{Z_{t+2}} t_{t+2}(z, \eta_{t+2}) R_{t+3}(z) d\mu}{(1 + g_{t+3}) \sum_{j=5}^6 sh_{j,t+3}}, \\
p_{t+4} &= \frac{\int_{Z_{t+3}} t_{t+3}(z, \eta_{t+3}) R_{t+4}(z) d\mu}{(1 + g_{t+4}) \sum_{j=5}^6 sh_{j,t+4}},
\end{aligned}$$

where $t_t(z, \eta)$, the amounts invested by the social security fund across producers must sum up to

the total tax revenues²²:

$$\begin{aligned}\int_{Z_{t+2}} t_{t+2}(z, \eta_{t+2}) d\mu &= \tau w_{t+2} \sum_{j=2}^4 sh_{j,t+2}(1 + \pi_j), \\ \int_{Z_{t+3}} t_{t+3}(z, \eta_{t+3}) d\mu &= \tau w_{t+3} \sum_{j=2}^4 sh_{j,t+3}(1 + \pi_j).\end{aligned}$$

I assume the same expectation formation mechanism as in the benchmark economy. As a result, note that given expectations for future wages and interest rates, the household problem is exactly the same. In particular, marginal propensities to consume and save are the same as in the benchmark economy, and largely the result of changes in demographic characteristics: survival probabilities and population growth rates.

Once the representative firm has paid the entry cost, the output produced when K_t units of capital and N_t units of labor are used is given by the following schedule:

$$Y(K_t, N_t; z(\eta_t)) = z(\eta_t) K_t^\alpha N_t^{1-\alpha},$$

where $\eta \in \{L, H\}$, $0 < \alpha < 1$ and $0 < z_L < z_H < 1$. After the aggregate shock is realized and conditional on paying the entry cost and on using capital K_t , the representative producer takes the wage rate w_t as given and chooses its labor input by solving:

$$\Pi(K_t, w_t; z(\eta_t)) \equiv \max_{N_t > 0} Y(K_t, N_t; z(\eta_t)) - w_t N_t.$$

Like in the benchmark economy, the representative producer cannot self-finance and needs to borrow from households. To this end each project issues state-contingent securities that pay $(x(L), x(H)) \geq 0$ in each state of the world, chooses capital K_t , and takes the households willingness to pay for these securities, $q_t(x_t(L), x_t(H))$ as given, such as to solve:

²²Although I technically no longer need tax revenues to be invested for one period before being disbursed as social security benefits, I keep this for comparability purposes with the benchmark

$$c_{y,t} + \epsilon E(c_{o,t+1}(\eta) | \eta_{t-1})$$

subject to

$$c_{y,t} \leq q_t(x_t(L), x_t(H)) - K_t - N_t e,$$

$$c_{o,t+1}(L) \leq \Pi(K_t, w_t(L); z_L) - x_t(L),$$

$$c_{o,t+1}(H) \leq \Pi(K_t, w_t(H); z_H) - x_t(H),$$

where the only difference relative to the benchmark is the fact that producers issue at most one security type and therefore do not pay security creation costs.

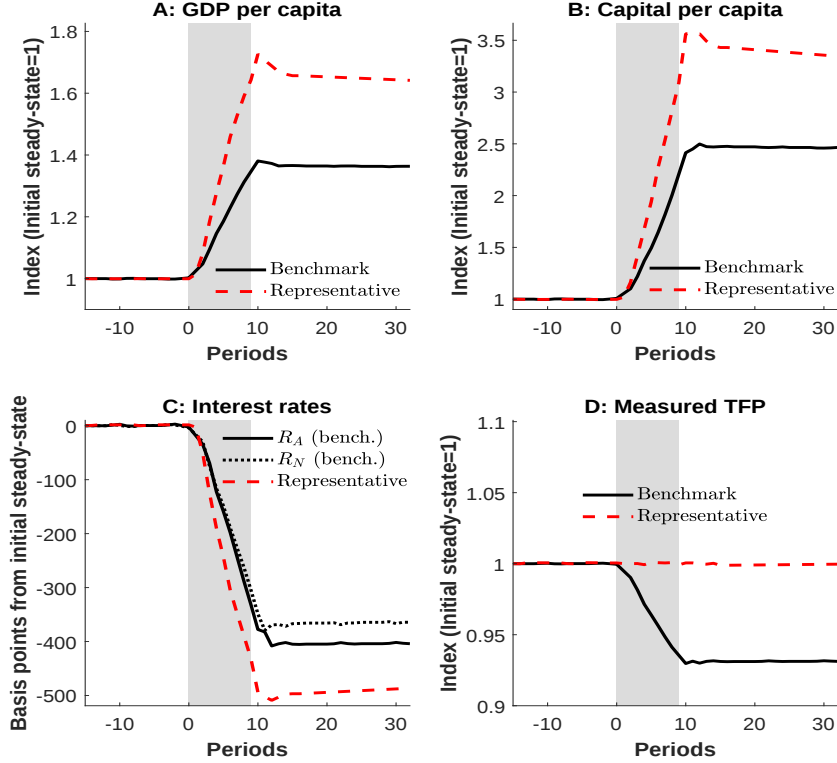
3.3.1 A demographic transition

I simulate the same demographic transition as before, subject only to the changes highlighted in the previous section. The model's calibration is much simpler given the reduced number of parameters. The discipline is the same though, as the values of the remaining parameters are chose to match the targets in Table 3. There is one exception: because the modified model only features one interest rate, I set β such that the capital-output ratio in the final steady-state is the same as in the benchmark economy.²³

The comparison between the two economies is made plain in Figure 6. Even though the driving force behind the transition is the same, capital used in production (and consequently household saving) is much larger (panel B) and the interest rate lower (panel C) than in the benchmark economy. The main reason for the difference should now be apparent. In the representative firm economy, the schedule for the supply of funds increases (a direct consequence of the demographic transition) and slides down along a static capital demand schedule, lowering the interest rate. In contrast, in the benchmark economy, this downward pressure on interest rates coming from the increase in the supply of funds is partially *counteracted* by a change in financing policies at the project level and by the activation of further projects (detailed in the previous section) that shift

²³I also simulated a version where I set β such that the interest rate in the final steady-state is equal to the (asset weighted) average of the two interest rates in the final steady-state of the benchmark model and the results differ very little.

Figure 6: Comparison to representative firm, one asset, economy



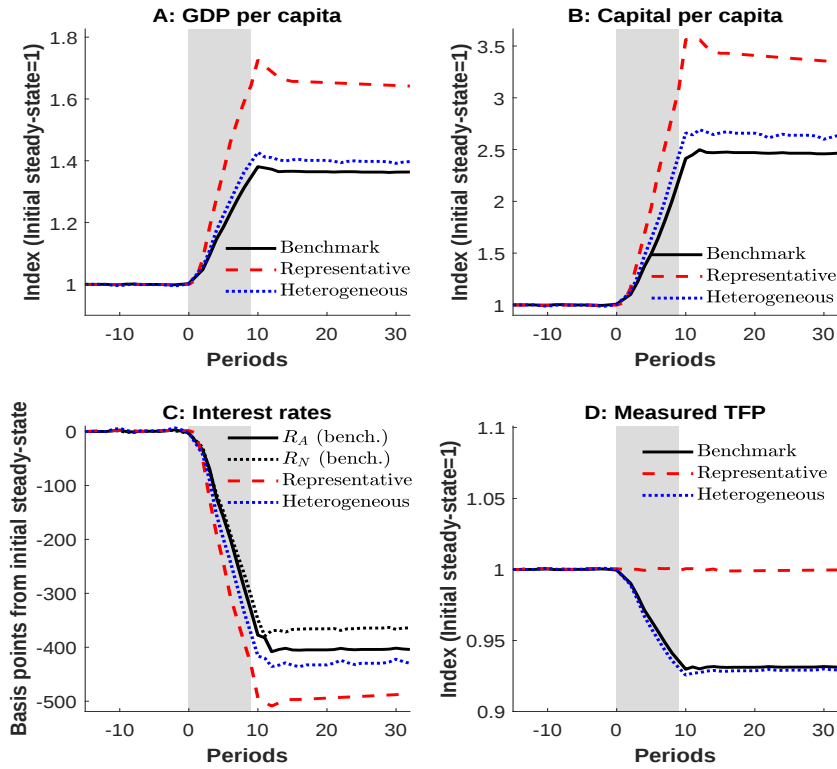
the capital demand schedule up, resulting in a smaller interest rate decrease.

These results imply that the effect of the demographic transition on output, the *demographic dividend*, can be severely overestimated by standard representative firm models, as panel A in Figure 6 makes clear: an output increase of 64 percent versus 36 percent in the benchmark. In yearly terms this is a growth rate of 0.39 percent – compared to 0.24 percent in the benchmark, making for an overestimation larger than 50 percent. As we previously discussed, part of the reason for the difference is the endogenous entry of new projects. Unlike what happens in the representative firm economy where by definition TFP is constant, in the benchmark economy the average project productivity goes down, as shown in panel D of Figure 6. Prior to the transition, these new entrants were infra-marginal projects who start to operate once interest rates start dropping, making their production plans profitable enough to cover the entry cost.

In comparing the representative firm economy and the benchmark, it is instructive to separate the changes caused by the introduction of heterogeneous producers – which may affect asset issuance

through entering and exiting – from those that arise from the introduction of an extra (riskless) asset – which change the asset issuance mix projects use to finance production. In order to do this decomposition I simulate another version of the economy where producers are heterogeneous in the same way as in the benchmark, but there is just one (state-contingent) asset and all household cohorts have the same preferences, like in the representative firm economy. In this intermediate economy projects can entry and exit but cannot finance themselves by issuing different assets.²⁴

Figure 7: **Decomposing the difference**

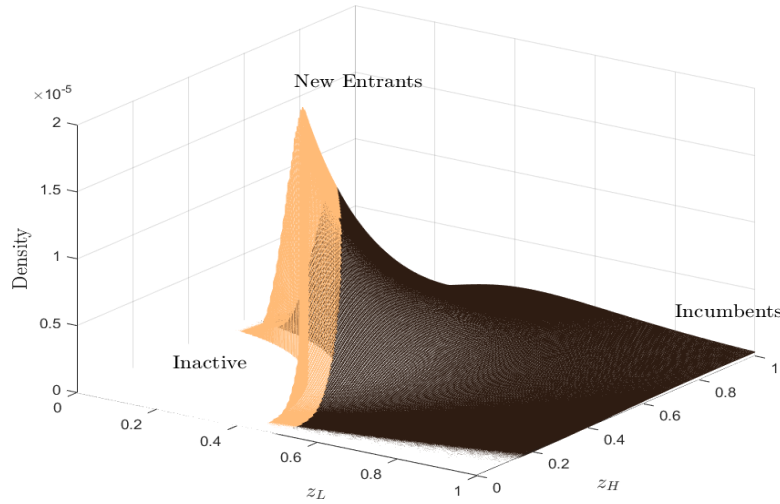


The results in Figure 7 make clear that the bulk of the differences are coming about because of the skill heterogeneity assumption, but that the ability to switch financing sources plays a non-negligible role. The latter accounts for about one third of the difference in the interest rate decline between the benchmark economy and the representative firm economy (panel B), and roughly one fifth of the

²⁴This economy's parameters are calibrated so as to match the targets in Table 3 except for the interest rate targets. Instead, the discount factor β is calibrated so that the capital-output ratio in the final-steady-state is the same as in the benchmark economy (which is also the same as in the representative firm economy, for that matter).

difference in the growth of capital (panel C). Differences in measured TFP are rooted entirely in the heterogeneity assumption (panel D). As explained above, projects that were unprofitable before the transition, find it worthwhile to pay the entry cost and operate once interest rates drop. From the original steady-state of the benchmark economy to the final one, the measure of active projects grows by more than a third. These projects are, on average, less productive than the incumbent operating projects, thus lowering measured TFP. Figure 8 shows this process: the darker density corresponds to projects that were active in the initial-steady-state, while the lighter density corresponds to the additional projects that are also active in the final steady-state.

Figure 8: **Growth in active projects**



Finally, the benchmark also differs from the representative firm economy in the sense that issuance is costly when both assets are issued at the same time. I also simulated a demographic transition under the assumption that these issuance costs are zero. When this is the case, all projects issue both asset types, and while such an economy is slightly different from the benchmark economy in levels, the changes along the transition show no significant difference.²⁵

²⁵While I omitted this experiment's results for the sake of brevity, they are available on request.

3.4 Discussion

In the benchmark economy projects can issue both contingent and non-contingent bonds. One objection that may arise in mapping from model to data is that developing economies are, for the most part, characterized by inchoate capital markets where the vast majority of firms do not issue corporate bonds, let alone highly-rated ones. Here, it is important to note that for the main model mechanism to work, it need not be the case that asset issuance is done by producers. In particular, the model would be isomorphic to one where, because of information asymmetries between households and producers, fully informed intermediaries with market power would take household savings and use them to buy a pool of projects, driving entrepreneur profit to zero (their outside cost). These intermediaries would then offer households a menu of state-contingent and non-contingent securities, just like in the current version of the model. The only difference would be that what we now call producer profits would constitute instead financial sector profits. In this case, non-contingent assets would simply be bank deposits, while the contingent securities would be equity in banks. Alternatively still, intermediaries would not even have to acquire projects, and they could simply serve as fully competitive pass-through entities (making zero profits) between households and projects. In this case, the savings of the older cohorts would be akin to bank loans, while the savings of the younger cohorts would be akin to equity. In any case, the alternative types of financing described here, would be compatible with the existing level of financial sophistication in developing economies, while not changing any of the results.

Another issue that is worth discussing is the fact that for a model that emphasizes safe asset supply, short-term government securities – the securities that epitomize the safe asset class, at least in developed economies – are conspicuously absent. The implication being that the share of household savings that are tied up in government securities would not be available for business financing, and therefore the model could be overestimating the amount of safe business financing. In defense of the model, such a simplification – mapping savings in the form of government securities in the data, to savings used as productive capital in the model – is a common one in macro models for a good reason: it is cumbersome to model simultaneous holdings of different securities with a common expected return absent other security characteristics like liquidity, for example. Moreover, to the extent that this type of government funding eventually ends up financing downstream projects much

in the same way, then little is lost in the abstraction.

4 Conclusion

The economies of West Africa are home to some of the youngest populations in the world. Over the next decades, as their demographic transition process continues, the share of working-age people is estimated to grow, giving rise to important macroeconomic changes. The model in this study is designed to capture some of these changes like the effects coming from age-specific labor productivity, those coming from changes in household saving rates and portfolio changes, as well as changes in asset issuance. The model predicts that the effect of the demographic transition on the output per capita of such developing economies is in the order of a quarter of a percentage point a year.

Asset markets are a key transmission mechanism to understand the macroeconomic impact of demographic changes. As such, it stands to reason that the quantitative evaluation of such impact should depend on the modeling assumptions surrounding said asset markets. I argue that omitting (i) producer heterogeneity with respect to productivity in different states of the world and (ii) producers' ability to finance themselves by issuing different types of assets, gives rise to quantitatively important differences when evaluating a demographic transition's impact. In particular, such models overestimate the impact on output and the size of the accompanying interest rate reduction.

While the analysis in this paper is done in the context of an economy that starts out as relatively young, the same should be true of industrialized economies that have been debating the economic consequences of their aging population. Economies like Japan, Italy, or Portugal, for example, exhibit total fertility rates in the order of 1.3-1.4 births per woman, well below replacement levels. True, matters are not as extreme in other developed economies, but even there, the concern around an aging population has generated a burgeoning literature investigating the impact of demographics on interest rates in industrialized economies.²⁶ In future work, I plan to understand whether introducing a business asset supply channel significantly changes the models' predictions regarding predicted decreases in interest rates in advanced economies.

²⁶See, for examples, [Lisack, Sajedi, and Thwaites \(2017\)](#) and [Carvalho, Ferrero, and Nechio \(2016\)](#).

A Data

The population data are from the United Nation’s *World Population Prospects*. The data were obtained February 27, 2020 from <https://population.un.org/wpp/Download/Standard/Population/>. In particular, I use File POP/7-1 that contains total population (both sexes combined) by five-year age groups, at the region, subregion and country levels from 1950-2100. From 2020 on, the predictions come in different variants, and I use the *no-migration* variant, as the model is not built to handle migration.

For each region, I have a panel with the population in each age group 0-4, 5-9, ..., 100+ every five years from 1950 to 2100. I need to regroup these 5-year cohorts into the model’s 16-year cohorts. I proceed in 3 steps:

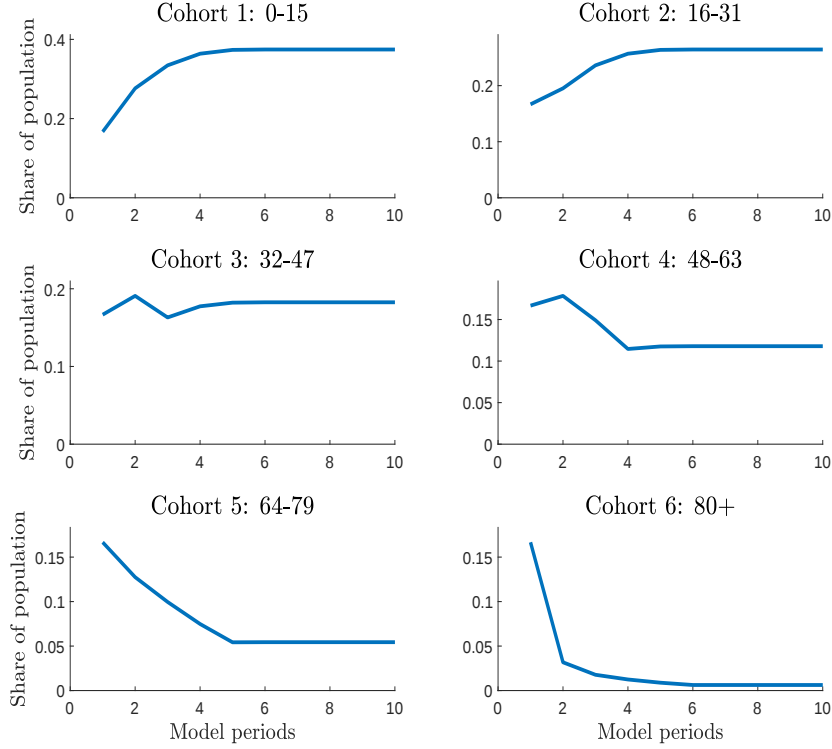
1. Split each 5-year cohort’s population into individual years of age. Instead of assuming a uniform distribution within cohorts, I assume that population density by age within each cohort moves linearly with the same gradient as two consecutive 5-year population cohorts. So, just for illustrative purposes, if cohort 0-4 has 100 people and cohort 5-9 has 90 people, I assume that the age distribution within the first cohort is such that population declines by 10 percent each year, and so I have from ages 0 to 4: 24.42, 21.98, 19.78, 17.80, and 16.02. This leaves me with a panel of 31 – five year periods (from 1950 to 2100) – by 101 – ages (from 0 to 100+).
2. I obtain population by age estimates for all years, not just every five years, by assuming linearity in the evolution of each age cohort. So if in year t there are 100 people age 10 and in year $t + 5$ there are 110 people aged 10, this means there will be 102 people aged 10 in year $t + 1$, 104 people aged 10 in year $t + 2$, etc. This leaves me with a panel of 151 years by 101 ages.
3. I regroup the different ages into 6, 16-year cohorts: 0-15, 16-31, 32-47, 48-63, 64-79, and 80+, where the last cohort is special in that it contains a fraction of people older than 95, albeit not a large one. Finally, since the model period lasts 16 years, starting in 2020 I take a cross section of these cohorts every 16 years.

Table 4: West-Africa model-compatible population (thousands)

Years/Cohorts	0-15	16-31	32-47	48-63	64-79	80+
2020	182,527	110,789	63,979	32,082	11,492	993
2036	245,295	173,211	102,820	55,514	19,884	1,775
2052	303,184	236,461	163,528	91,564	36,185	3,606
2068	344,161	295,116	226,094	147,940	62,440	7,472
2084	368,312	337,088	284,545	207,157	10,4391	14,811
2100	373,926	362,254	326,912	263,555	151,251	27,868

I take West-Africa as a benchmark for the initial model steady-state. This includes Benin, Burkina-Faso, Cape Verde, Ivory Coast, Gambia, Ghana, Guinea, Guinea-Bissau, Liberia, Mali,

Figure 9: **Initial steady-state (West-Africa)**



Mauritania, Niger, Nigeria, Senegal, Sierra Leone, and Togo. The model-compatible cohort populations are shown in Table 4.

I use the changes from 2020 to 2036 to compute model-compatible population growth and survival rates that I can then use to create a synthetic steady-state. The population growth rate in the model's steady-state is the growth rate in the younger cohort's size from one period to another, which in this case is $n = \frac{245,300}{182,530} - 1 = 0.3439$, or 1.86% in yearly terms. Letting $N_{j,t}$ denote the population of cohort $j = 1, \dots, 5$ in period t , survival probabilities for cohort j are given by $\frac{N_{j+1,t+1}}{N_{j,t}}$, which for $t = 2020$ are $s = (0.9490, 0.9281, 0.8677, 0.6198, 0.1545)$.

Next, I compute the steady-state cohort shares that these parameters generate. I do this by simulation, and start by setting population cohort shares to an arbitrary level: $P(j, t) = 1/6$, and compute the shares recursively for a large enough number of periods T according to:

$$\begin{aligned} P(1, t+1) &= (1+n)P(1, t) \\ P(j+1, t+1) &= s_j P(j, t) \quad \text{for } j = 1, \dots, 5. \end{aligned}$$

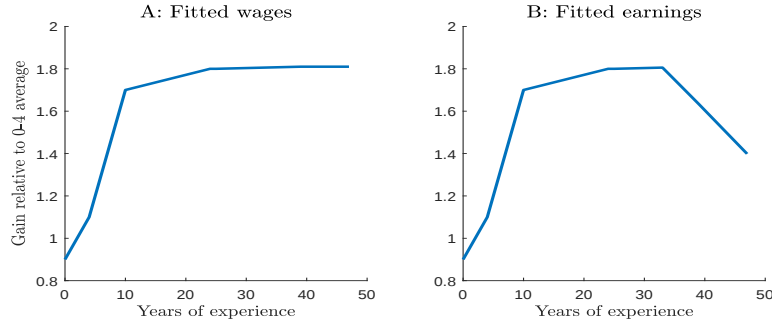
As Figure 9 shows, for the West Africa case, the convergence in shares is very quickly. The resulting steady-state shares are used in the model for the initial steady-state and show up in Table 2.

B Lifetime earnings profiles

I normalize the human capital of young adults to $\pi_2 = 1$ implying their wage earnings are w and assume that human capital accumulation throughout an agent's lifetime evolves exogenously and in such a way that prime-aged wages are $\pi_3 w$ and middle-aged wages are $\pi_4 w$. I then calibrate π_3 and π_4 so that the earnings gains in the model match the earnings gains implied by the findings of [Rupert and Zanella \(2015\)](#) and [Lagakos, Moll, Porzio, Qian, and Schoellman \(2018\)](#). Here I provide some more details about this calibration.

[Lagakos, Moll, Porzio, Qian, and Schoellman \(2018\)](#) estimate experience-wage profiles for a set of rich and poor countries. They are able to disentangle experience, time, and cohort effects by assuming that there should be no experience effects on wages near the end of the life-cycle when the incentives to invest in human capital, or look for better jobs, are much smaller than earlier in life. Under the assumption that there are no experience effects in the last 10 years of potential experience and no depreciation, they report (see their Table 4, panel A) that wage gains for the 5-9 experience group are 40.3% larger than for the 0-4 group, while gains are 79.3% for the 20-24 group and 80.8% for the 35-39 group. These results are for an average of 4 rich countries (United States, Canada, Germany and the United Kingdom). I will take them to reflect the U.S economy, as the data in [Lagakos, Moll, Porzio, Qian, and Schoellman \(2018\)](#), suggest (see their Figure 5, for example) that the U.S. is close to the average.

Figure 10: **Life-cycle wages and earnings**



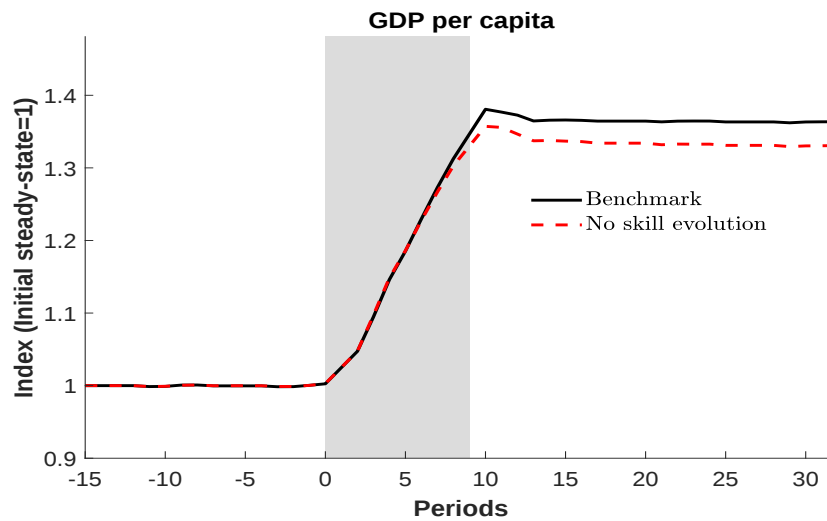
I use a piecewise linear function of weakly increasing wages to match these 3 relative differences. This is shown in panel A of Figure 10. Since the model does not have an extensive margin (workers supply labor inelastically), I map model's labor compensation to earnings. In order to convert the above wage profile to earnings I use the results in [Rupert and Zanella \(2015\)](#). They find that hours worked do not vary with age up until age 50 (in a statistical sense). This means that life-cycle earnings track life-cycle wages up until age 50. From then on, workers significantly (and statistically) reduce their working hours in anticipation of retirement. Figure 6 in [Rupert and Zanella \(2015\)](#) suggests that yearly hours decline from about 2200 to 1700. I assume this decline occurs linearly for the last 15 years of a worker's career. This results in the earnings pattern shown in panel B of Figure 10.

Using this earnings profile I compute the average earnings for each of the three 16-year periods of a worker's career and find that prime-aged workers (16-31 years of experience) earn 28% more than young workers (0-15 years of experience), while middle aged workers (32-47 years of experience) earn 16% more than young workers. I therefore set $\pi_3 = 0.28$ and $\pi_4 = 0.16$.

B.1 No lifetime worker productivity changes

To gauge the importance of modeling lifetime skill changes, I rerun the demographic transition assuming that workers are equally productivity throughout their lifetime. In terms of the model's parameters, this simply means setting $\pi_3 = \pi_4 = 1$. As Figure 11 shows, there are hardly any differences in output per capita during the transition and the steady-state difference amounts to less than ten percent.

Figure 11: **Robustness: no lifetime worker productivity changes**



C The household problem

In this section I solve the household problem and derive expressions for cohort-specific saving: equations (C.6).

These expressions are contingent on a set of expected variables whose calculation I make clear in the next section.

$$\max E_t \left\{ \gamma (1 + b_t) \log c_{1,t}(\eta_t) + \log c_{2,t}(\eta_t) + s_{2,t} \beta \log \{E_t (c_{3,t+1}(\eta_{t+1}) | \eta_t)\} + s_{3,t+1} \beta^2 \log \{E_{t+1} (c_{4,t+2}(\eta_{t+2}) | \eta_{t+1})\} \right. \\ \left. + s_{4,t+2} \beta^3 \log \{\min (c_{5,t+3}(L), c_{5,t+3}(H))\} + s_{5,t+3} \beta^4 \log \{\min (c_{6,t+4}(L), c_{6,t+4}(H))\} \right\}$$

$$\begin{aligned} s.t. \quad c_{1,t}(\eta_t) + c_{2,t}(\eta_t) + \int_{Z_t} a_{2,t}(z, \eta_t) d\mu &\leq (1 - \tau) w_t + b_{2,t}, \\ c_{3,t+1}(\eta_{t+1}) + \int_{Z_t} a_{3,t+1}(z, \eta_{t+1}) d\mu &\leq (1 - \tau)(1 + \pi_3) w_{t+1} + b_{3,t+1} + \int_{Z_t} a_{2,t}(z, \eta_t) R_{t+1}^R(z) d\mu, \\ c_{4,t+2}(\eta_{t+2}) + \int_{Z_t} a_{4,t+2}(z, \eta_{t+2}) d\mu &\leq (1 - \tau)(1 + \pi_4) w_{t+2} + b_{4,t+2} + \int_{Z_t} a_{3,t+1}(z, \eta_{t+1}) R_{t+2}^R(z) d\mu, \\ c_{5,t+3}(\eta_{t+3}) + \int_{Z_t} a_{5,t+3}(z, \eta_{t+3}) d\mu &\leq b_{5,t+3} + p_{t+3} + \int_{Z_t} a_{4,t+2}(z, \eta_{t+2}) R_{t+3}^S(z) d\mu, \\ c_{6,t+4}(\eta_{t+4}) &\leq p_{t+4} + \int_{Z_t} a_{5,t+3}(z, \eta_{t+3}) R_{t+4}^S(z) d\mu, \\ c_{1,t}, c_{2,t}, c_{3,t+1}, c_{4,t+2}, c_{5,t+3}, c_{6,t+4} &> 0, \end{aligned}$$

where the incidental bequests are given by

$$\begin{aligned} b_{2,t} &= \frac{sh_{2,t-1}}{sh_{2,t}} \frac{1 - s_{2,t-1}}{1 + g_t} \int_{Z_t} a_{2,t-1}(z) R_t^R(z) d\mu, \\ b_{3,t+1} &= \frac{sh_{3,t}}{sh_{3,t+1}} \frac{1 - s_{3,t}}{1 + g_{t+1}} \int_{Z_t} a_{3,t}(z) R_{t+1}^R(z) d\mu, \\ b_{4,t+2} &= \frac{sh_{4,t+1}}{sh_{4,t+2}} \frac{1 - s_{4,t+1}}{1 + g_{t+2}} \int_{Z_t} a_{4,t+1}(z, \eta_{t+1}) R_{t+2}^S(z) d\mu, \\ b_{5,t+3} &= \frac{sh_{5,t+2}}{sh_{5,t+3}} \frac{1 - s_{5,t+2}}{1 + g_{t+3}} \int_{Z_t} a_{5,t+2}(z, \eta_{t+2}) R_{t+3}^S(z) d\mu, \end{aligned}$$

and the social security benefits are given by:

$$\begin{aligned} p_{t+3} &= \frac{\int_{Z_t} t_{t+2}(z, \eta_{t+2}) R_{t+3}^S(z) d\mu}{(1 + g_{t+3}) \sum_{j=5}^6 sh_{j,t+3}}, \\ p_{t+4} &= \frac{\int_{Z_t} t_{t+3}(z, \eta_{t+3}) R_{t+4}^S(z) d\mu}{(1 + g_{t+4}) \sum_{j=5}^6 sh_{j,t+4}} \end{aligned}$$

where $t_t(z, \eta)$, the amounts invested by the social security fund across producers that issue safe securities must sum up to the total tax revenues:

$$\int_{Z_t} t_{t+2}(z, \eta_{t+2}) d\mu = \tau w_{t+2} \sum_{j=2}^4 sh_{j,t+2}(1 + \pi_j),$$

$$\int_{Z_t} t_{t+3}(z, \eta_{t+3}) d\mu = \tau w_{t+3} \sum_{j=2}^4 sh_{j,t+3}(1 + \pi_j).$$

In what follows I will abuse notation and introduce some equilibrium results in order to make the notation lighter. In particular, let $R_t^S = R_t^S(z)$ and $R_t^R = R_t^R(z)$ for all active projects $z \in Z$ denote, respectively, the equilibrium safe and risky rates, and let $a_{j,t} = \int_{Z_t} a_{j,t}(z) d\mu$ denote total savings of cohort j . I will also write $\underline{c}_{5,t+3} = \min(c_{5,t+3}(L), c_{5,t+3}(H))$, $\underline{c}_{6,t+4} = \min(c_{6,t+4}(L), c_{6,t+4}(H))$ for the, riskless, consumption of infinitely risk-averse cohorts.

The first-order conditions are

$$\frac{c_{1,t}}{c_{2,t}} = \gamma(1 + b_t) \quad (\text{C.1})$$

$$\frac{E_t c_{3,t+1}}{c_{2,t}} = \beta s_{2,t} E_t R_{t+1}^R \quad (\text{C.2})$$

$$\frac{E_t c_{4,t+2}}{E_t c_{3,t+1}} = \beta \frac{s_{3,t+1}}{s_{2,t}} E_t R_{t+2}^R \quad (\text{C.3})$$

$$\frac{E_t \underline{c}_{5,t+3}}{E_t c_{4,t+2}} = \beta \frac{s_{4,t+2}}{s_{3,t+1}} E_t R_{t+3}^S \quad (\text{C.4})$$

$$\frac{E_t \underline{c}_{6,t+4}}{E_t \underline{c}_{5,t+3}} = \beta \frac{s_{5,t+3}}{s_{4,t+2}} E_t R_{t+4}^S, \quad (\text{C.5})$$

for all t and for all aggregate states $\eta_t \in \{L, H\}$.

Note that the reason we can write the FOC (C.3) as $\frac{E_t c_{3,t+1}}{c_{2,t}} = \beta s_{2,t} E_t R_{t+1}^R$, instead of the usual $\frac{1}{c_{2,t}} = \beta s_{2,t} E_t \left(\frac{R_{t+1}^R}{c_{3,t+1}} \right)$, is that the utility argument is $E_t c_{3,t+1}$ and not $c_{3,t+1}$. This means the FOC is $\frac{1}{c_{2,t}} = \beta s_{2,t} E_t \left(\frac{R_{t+1}^R}{E_t c_{3,t+1}} \right)$, which means that conditional on time t information, $E_t c_{3,t+1}$ is a constant and can, therefore, be brought out of the expectation: $E_t \left(\frac{R_{t+1}^R}{E_t c_{3,t+1}} \right) = \frac{1}{E_t c_{3,t+1}} E_t R_{t+1}^R$.

Putting ourselves in the perspective of a household in the last period of its life: $c_{6,t+4} = p_{t+4} + a_{5,t+3} R_{t+4}^S$, for all η_{t+4} . Abusing notation and using the equilibrium result that $c_{6,t+4} = \underline{c}_{6,t+4}$, and $c_{5,t+3} = \underline{c}_{5,t+3}$, we can use the analogous of FOC (C.5) in period $t+3$ to substitute out $c_{6,t+4}$:

$$\frac{E_{t+3} p_{t+4} + a_{5,t+3} E_{t+3} R_{t+4}^S}{c_{5,t+3}} = \beta \frac{s_{5,t+3}}{s_{4,t+2}} E_{t+3} R_{t+4}^S.$$

Further substituting out $c_{5,t+3} = p_{t+3} + a_{4,t+2} R_{t+3}^S + b_{t+3} - a_{5,t+3}$ from the early retirees budget constraint:

$$E_{t+3}p_{t+4} + a_{5,t+3}E_{t+3}R_{t+4}^S = \beta \frac{s_{5,t+3}}{s_{4,t+2}} E_{t+3}R_{t+4}^S (p_{t+3} + a_{4,t+2}R_{t+3}^S + b_{t+3} - a_{5,t+3}).$$

Solving for the saving of the early-retiree cohort:

$$a_{5,t+3} = \frac{\beta \frac{s_{5,t+3}}{s_{4,t+2}} (p_{t+3} + b_{5,t+3} + a_{4,t+2}R_{t+3}^S) - \frac{E_{t+3}p_{t+4}}{E_{t+3}R_{t+4}^S}}{1 + \beta \frac{s_{5,t+3}}{s_{4,t+2}}}. \quad (\text{C.6})$$

Letting $d_{5,t+3} \equiv \beta \frac{s_{5,t+3}}{s_{4,t+2}}$, note how optimal savings are a fraction $\frac{d_{5,t+3}}{1+d_{5,t+3}}$ of current income minus discounted future income.

We can now replace this optimal solution back in the early retirees budget constraint and solve for consumption:

$$c_{5,t+3} = p_{t+3} + a_{4,t+2}R_{t+3}^S + b_{t+3} - \frac{d_{5,t+3}}{1 + d_{5,t+3}} (p_{t+3} + b_{5,t+3} + a_{4,t+2}R_{t+3}^S) - \frac{\frac{E_{t+3}p_{t+4}}{E_{t+3}R_{t+4}^S}}{1 + d_{5,t+3}}$$

$$c_{5,t+3} = \left(\frac{1}{1 + d_{5,t+3}} \right) \left(p_{t+3} + \frac{E_{t+3}p_{t+4}}{E_{t+3}R_{t+4}^S} + a_{4,t+2}R_{t+3}^S + b_{5,t+3} \right),$$

Replacing the expression for the optimal $c_{5,t+3}$ we just found, as well as the expression for $c_{4,t+2}$ from the pre-retiree cohort's budget constraint, into the analogue of FOC (C.4) for the pre-retirees in period $t + 2$ we get:

$$E_{t+2} \left\{ \left(\frac{1}{1 + \beta \frac{s_{5,t+3}}{s_{4,t+2}}} \right) \left(\frac{p_{t+3}}{E_{t+2}R_{t+3}^S} + \frac{E_{t+3}p_{t+4}}{E_{t+2}R_{t+3}^S E_{t+3}R_{t+4}^S} + \frac{a_{4,t+2}R_{t+3}^S}{E_{t+2}R_{t+3}^S} + \frac{b_{5,t+3}}{E_{t+2}R_{t+3}^S} \right) \right\}$$

$$= \beta \frac{s_{4,t+2}}{s_{3,t+1}} ((1 - \tau)(1 + \pi_4)w_{t+2} + b_{4,t+2} + a_{3,t+1}R_{t+2}^R - a_{4,t+2})$$

Using the law of iterated expectations and recalling that households know the full evolution of the survival probabilities, $s_{j,t}$, which can, therefore, be brought out of the expectation, we get:

$$\left(\frac{1}{1 + \beta \frac{s_{5,t+3}}{s_{4,t+2}}} + \beta \frac{s_{4,t+2}}{s_{3,t+1}} \right) a_{4,t+2} = \beta \frac{s_{4,t+2}}{s_{3,t+1}} ((1 - \tau)(1 + \pi_4)w_{t+2} + b_{4,t+2} + a_{3,t+1}R_{t+2}^R)$$

$$- \frac{1}{1 + \beta \frac{s_{5,t+3}}{s_{4,t+2}}} \left(\frac{E_{t+2}p_{t+3}}{E_{t+2}R_{t+3}^S} + \frac{E_{t+2}p_{t+4}}{E_{t+2}R_{t+3}^S E_{t+2}R_{t+4}^S} + \frac{E_{t+2}b_{5,t+3}}{E_{t+2}R_{t+3}^S} \right)$$

Letting $d_{4,t+2} \equiv \beta \frac{s_{4,t+2}}{s_{3,t+1}} + \beta^2 \frac{s_{5,t+3}}{s_{3,t+1}}$ we get:

$$a_{4,t+2} = \frac{d_{4,t+2}}{1 + d_{4,t+2}} ((1 - \tau)(1 + \pi_4)w_{t+2} + b_{4,t+2} + a_{3,t+1}R_{t+2}^R)$$

$$- \frac{1}{1 + d_{4,t+2}} \left(\frac{E_{t+2}p_{t+3}}{E_{t+2}R_{t+3}^S} + \frac{E_{t+2}p_{t+4}}{E_{t+2}R_{t+3}^S E_{t+2}R_{t+4}^S} + \frac{E_{t+2}b_{5,t+3}}{E_{t+2}R_{t+3}^S} \right) \quad (\text{C.7})$$

We can now get consumption $c_{4,t+2}$ from the pre-retiree cohort's budget constraint:

$$c_{4,t+2} = \left(1 - \frac{d_{4,t+2}}{1 + d_{4,t+2}}\right) \left((1 - \tau)(1 + \pi_4)w_{t+2} + b_{4,t+2} + a_{3,t+1}R_{t+2}^R\right) + \frac{\frac{E_{t+2}p_{t+3}}{E_{t+2}R_{t+3}^S} + \frac{E_{t+2}p_{t+4}}{E_{t+2}R_{t+3}^S E_{t+2}R_{t+4}^S} + \frac{E_{t+2}b_{5,t+3}}{E_{t+2}R_{t+3}^S}}{1 + d_{4,t+2}}$$

$$c_{4,t+2} = \frac{(1 - \tau)(1 + \pi_4)w_{t+2} + b_{4,t+2} + a_{3,t+1}R_{t+2}^R + \frac{E_{t+2}p_{t+3}}{E_{t+2}R_{t+3}^S} + \frac{E_{t+2}p_{t+4}}{E_{t+2}R_{t+3}^S E_{t+2}R_{t+4}^S} + \frac{E_{t+2}b_{5,t+3}}{E_{t+2}R_{t+3}^S}}{1 + d_{4,t+2}}$$

Continuing to go backwards, we now stand in period $t+1$ with a household that is in their second period of work, with information set $\mathcal{I}_{t+1}$. Replacing the expression for the optimal $c_{4,t+2}$ we just found, as well as the expression for $c_{3,t+1}$ from the prime-age cohort's budget constraint, into FOC (C.3) we get:

$$E_{t+1} \left\{ \frac{(1 - \tau)(1 + \pi_4)w_{t+2} + b_{4,t+2} + a_{3,t+1}R_{t+2}^R + \frac{p_{t+3}}{R_{t+3}^S} + \frac{p_{t+4}}{R_{t+3}^S R_{t+4}^S} + \frac{b_{5,t+3}}{R_{t+3}^S}}{1 + d_{4,t+2}} \right\}$$

$$= \beta \frac{s_{3,t+1}}{s_{2,t}} E_{t+1} R_{t+2}^R [(1 - \tau)(1 + \pi_3)w_{t+1} + b_{3,t+1} + a_{2,t}R_{t+1}^R - a_{3,t+1}]$$

$$\frac{(1 - \tau)(1 + \pi_4) \frac{E_{t+1}w_{t+2}}{E_{t+1}R_{t+2}^R} + \frac{E_{t+1}b_{4,t+2}}{E_{t+1}R_{t+2}^R} + a_{3,t+1} + \frac{E_{t+1}p_{t+3}}{E_{t+1}R_{t+2}^R E_{t+1}R_{t+3}^S} + \frac{E_{t+1}p_{t+4}}{E_{t+1}R_{t+2}^R E_{t+1}R_{t+3}^S E_{t+1}R_{t+4}^S} + \frac{E_{t+1}b_{5,t+3}}{E_{t+1}R_{t+2}^R E_{t+1}R_{t+3}^S}}{1 + d_{4,t+2}}$$

$$= \beta \frac{s_{3,t+1}}{s_{2,t}} [(1 - \tau)(1 + \pi_3)w_{t+1} + b_{3,t+1} + a_{2,t}R_{t+1}^R - a_{3,t+1}]$$

Solving for $a_{3,t+1}$

$$a_{3,t+1} \left(\frac{1}{1 + d_{4,t+2}} + \beta \frac{s_{3,t+1}}{s_{2,t}} \right) = \beta \frac{s_{3,t+1}}{s_{2,t}} [(1 - \tau)(1 + \pi_3)w_{t+1} + b_{3,t+1} + a_{2,t}R_{t+1}^R]$$

$$= \frac{(1 - \tau)(1 + \pi_4) \frac{E_{t+1}w_{t+2}}{E_{t+1}R_{t+2}^R} + \frac{E_{t+1}b_{4,t+2}}{E_{t+1}R_{t+2}^R} + a_{3,t+1} + \frac{E_{t+1}p_{t+3}}{E_{t+1}R_{t+2}^R E_{t+1}R_{t+3}^S} + \frac{E_{t+1}p_{t+4}}{E_{t+1}R_{t+2}^R E_{t+1}R_{t+3}^S E_{t+1}R_{t+4}^S} + \frac{E_{t+1}b_{5,t+3}}{E_{t+1}R_{t+2}^R E_{t+1}R_{t+3}^S}}{1 + d_{4,t+2}}$$

Note that $\frac{1}{1 + d_{4,t+2}} + \beta \frac{s_{3,t+1}}{s_{2,t}} = \frac{1}{1 + \beta \frac{s_{4,t+2}}{s_{3,t+1}} + \beta^2 \frac{s_{5,t+3}}{s_{3,t+1}}} + \beta \frac{s_{3,t+1}}{s_{2,t}} = \frac{\beta \frac{s_{3,t+1}}{s_{2,t}} + \beta^2 \frac{s_{4,t+2}}{s_{2,t}} + \beta^3 \frac{s_{5,t+3}}{s_{2,t}}}{1 + \beta \frac{s_{4,t+2}}{s_{3,t+1}} + \beta^2 \frac{s_{5,t+3}}{s_{3,t+1}}}$, and therefore, letting $d_{3,t+1} \equiv \beta \frac{s_{3,t+1}}{s_{2,t}} + \beta^2 \frac{s_{4,t+2}}{s_{2,t}} + \beta^3 \frac{s_{5,t+3}}{s_{2,t}}$

$$a_{3,t+1} = \frac{d_{3,t+1}}{1 + d_{3,t+1}} [(1 - \tau)(1 + \pi_3)w_{t+1} + b_{3,t+1} + a_{2,t}R_{t+1}^R] \quad (\text{C.8})$$

$$= \frac{(1 - \tau)(1 + \pi_4) \frac{E_{t+1}w_{t+2}}{E_{t+1}R_{t+2}^R} + \frac{E_{t+1}b_{4,t+2}}{E_{t+1}R_{t+2}^R} + \frac{E_{t+1}b_{5,t+3}}{E_{t+1}R_{t+2}^R E_{t+1}R_{t+3}^S} + \frac{E_{t+1}p_{t+3}}{E_{t+1}R_{t+2}^R E_{t+1}R_{t+3}^S} + \frac{E_{t+1}p_{t+4}}{E_{t+1}R_{t+2}^R E_{t+1}R_{t+3}^S E_{t+1}R_{t+4}^S}}{1 + d_{3,t+1}}$$

We can now get consumption $c_{3,t+1}$ from the prime-aged cohort's budget constraint:

$$c_{3,t+1} = \left(\frac{1}{1 + d_{3,t+1}} \right) \left((1 - \tau)(1 + \pi_3)w_{t+1} + b_{3,t+1} + a_{2,t}R_{t+1}^R + (1 - \tau)(1 + \pi_4) \frac{E_{t+1}w_{t+2}}{E_{t+1}R_{t+2}^R} \right. \\ \left. + \frac{E_{t+1}b_{4,t+2}}{E_{t+1}R_{t+2}^R} + \frac{E_{t+1}b_{5,t+3}}{E_{t+1}R_{t+2}^R E_{t+1}R_{t+3}^S} + \frac{E_{t+1}p_{t+3}}{E_{t+1}R_{t+2}^R E_{t+1}R_{t+3}^S} + \frac{E_{t+1}p_{t+4}}{E_{t+1}R_{t+2}^R E_{t+1}R_{t+3}^S E_{t+1}R_{t+4}^S} \right)$$

Continuing to go backwards, we now stand in period t with a household that is in their second period of work, with information set $\mathcal{I}_t$. Replacing the expression for the optimal $c_{3,t+1}$ we just found into FOC (C.2) we get:

$$\left(\frac{1}{\beta s_{2,t}(1 + d_{3,t+1})} \right) \left((1 - \tau)(1 + \pi_3) \frac{E_t w_{t+1}}{E_t R_{t+1}^R} + \frac{E_t b_{3,t+1}}{E_t R_{t+1}^R} + a_{2,t} + (1 - \tau)(1 + \pi_4) \frac{E_t w_{t+2}}{E_t R_{t+1}^R E_t R_{t+2}^R} \right. \\ \left. + \frac{E_t p_{t+3}}{E_t R_{t+1}^R E_t R_{t+2}^R E_t R_{t+3}^S} + \frac{E_t p_{t+4}}{E_t R_{t+1}^R E_t R_{t+2}^R E_t R_{t+3}^S E_t R_{t+4}^S} + \frac{E_t b_{4,t+2}}{E_t R_{t+1}^R E_t R_{t+2}^R} + \frac{E_t b_{5,t+3}}{E_t R_{t+1}^R E_t R_{t+2}^R E_t R_{t+3}^S} \right) = c_{2,t}$$

From the FOC C.1: $c_{1,t} = \gamma(1 + b_t)c_{2,t}$. Replacing this in the young adult cohort budget constraint: $a_{2,t} = (1 - \tau)w_t + b_{2,t} - (1 + \gamma(1 + b_t))c_{2,t}$. This gives us:

$$a_{2,t} = (1 - \tau)w_t + b_{2,t} - \left[\left(\frac{1 + \gamma(1 + b_t)}{\beta s_{2,t}(1 + d_{3,t+1})} \right) \left((1 - \tau)(1 + \pi_3) \frac{E_t w_{t+1}}{E_t R_{t+1}^R} + a_{2,t} + (1 - \tau)(1 + \pi_4) \frac{E_t w_{t+2}}{E_t R_{t+1}^R E_t R_{t+2}^R} \right. \right. \\ \left. \left. + \frac{E_t p_{t+3}}{E_t R_{t+1}^R E_t R_{t+2}^R E_t R_{t+3}^S} + \frac{E_t p_{t+4}}{E_t R_{t+1}^R E_t R_{t+2}^R E_t R_{t+3}^S E_t R_{t+4}^S} + \frac{E_t b_{3,t+1}}{E_t R_{t+1}^R} + \frac{E_t b_{4,t+2}}{E_t R_{t+1}^R E_t R_{t+2}^R} + \frac{E_t b_{5,t+3}}{E_t R_{t+1}^R E_t R_{t+2}^R E_t R_{t+3}^S} \right) \right]$$

Letting $d_{2,t} \equiv \beta s_{2,t}(1 + d_{3,t+1}) = \beta s_{2,t} + \beta^2 s_{3,t+1} + \beta^3 s_{4,t+2} + \beta^4 s_{5,t+3}$ and solving for $a_{2,t}$, we get:

$$a_{2,t} = \frac{d_{2,t} [(1 - \tau)w_t + b_{2,t}]}{1 + d_{2,t} + \gamma(1 + b_t)} - \frac{(1 - \tau)(1 + \pi_3) \frac{E_t w_{t+1}}{E_t R_{t+1}^R} + (1 - \tau)(1 + \pi_4) \frac{E_t w_{t+2}}{E_t R_{t+1}^R E_t R_{t+2}^R}}{1 + d_{2,t} + \gamma(1 + b_t)} \quad (C.9) \\ - \frac{\frac{E_t p_{t+3}}{E_t R_{t+1}^R E_t R_{t+2}^R E_t R_{t+3}^S} + \frac{E_t p_{t+4}}{E_t R_{t+1}^R E_t R_{t+2}^R E_t R_{t+3}^S E_t R_{t+4}^S} + \frac{E_t b_{3,t+1}}{E_t R_{t+1}^R} + \frac{E_t b_{4,t+2}}{E_t R_{t+1}^R E_t R_{t+2}^R} + \frac{E_t b_{5,t+3}}{E_t R_{t+1}^R E_t R_{t+2}^R E_t R_{t+3}^S}}{1 + d_{2,t} + \gamma(1 + b_t)}$$

Equipped with the expressions for cohort saving we can use these in solving for expectation formation.

D Expectations formation

At every period t , I solve for saving decisions $a_{2,t}$, $a_{3,t}$, $a_{4,t}$, and $a_{5,t}$ using equations (C.6)-(C.9) above. To do this I need instances for the following expectations: $E_t w_{t+1}$, $E_t R_{t+1}^R$, $E_t R_{t+1}^S$, $E_t p_{t+1}$, $E_t w_{t+2}$, $E_t R_{t+2}^R$, $E_t R_{t+2}^S$, $E_t p_{t+2}$, $E_t b_{5,t+2}$, $E_t b_{4,t+2}$, $E_t R_{t+3}^S$, $E_t p_{t+3}$, $E_t b_{5,t+3}$, $E_t R_{t+4}^S$, and $E_t p_{t+4}$. Notice that expectations on bequests and pensions depend only on future assets, interest rates and wage rates.

The assumption is that households have perfect foresight over the evolution of demographic characteristics and therefore know future cohort survival probabilities and $s_{j,t}$ and cohort shares $sh_{j,t}$ for $j = 1, \dots, 6$ and $t = 0, \dots$, and form expectations regarding next period's wages and interest rates by regressing these on a subset of their information set at time t containing the history of states $\mathcal{I}_t = \{\eta_0, \dots, \eta_{t-1}, a_0^R, \dots, a_{t-1}^R, a_0^S, \dots, a_{t-1}^S\}$. Since the economy's states in period t are $\{\eta_{t-1}, a_{t-1}^R, a_{t-1}^S\}$, the regressions are:

$$\begin{aligned} w_t &= \beta_0^w + \beta_\eta^w \eta_{t-1} + \beta_R^w a_{t-1}^R + \beta_S^w a_{t-1}^S + \varepsilon_t^w, \\ R_t^R &= \beta_0^R + \beta_\eta^R \eta_{t-1} + \beta_R^R a_{t-1}^R + \beta_S^R a_{t-1}^S + \varepsilon_t^R, \\ R_t^S &= \beta_0^S + \beta_\eta^S \eta_{t-1} + \beta_R^S a_{t-1}^R + \beta_S^S a_{t-1}^S + \varepsilon_t^S. \end{aligned}$$

This gives me estimated coefficients $\hat{\beta}_0^w$, $\hat{\beta}_\eta^w$, $\hat{\beta}_R^w$, $\hat{\beta}_S^w$, for the wage rate and its counterparts for the risky interest rate: $\hat{\beta}_0^R$, $\hat{\beta}_\eta^R$, $\hat{\beta}_R^R$, $\hat{\beta}_S^R$, and the safe interest rate $\hat{\beta}_0^S$, $\hat{\beta}_\eta^S$, $\hat{\beta}_R^S$, $\hat{\beta}_S^S$. I then use these coefficients and the actual states to compute a one-period-ahead forecast, which I assume are the households' expectations: $E_t w_{t+1}$, $E_t R_{t+1}^R$, and $E_t R_{t+1}^S$:

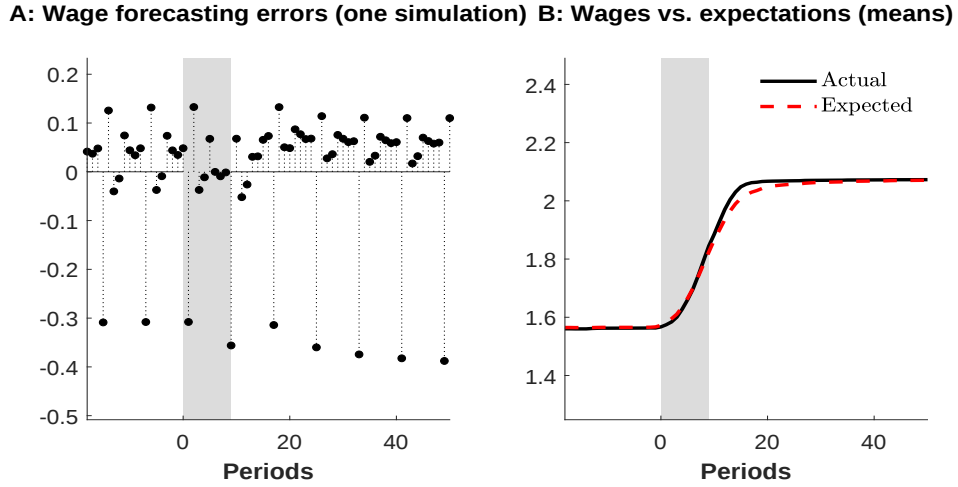
$$\begin{aligned} E_t w_{t+1} &\equiv \hat{\beta}_0^w + [M(L|\eta_{t-1})\eta(L) + M(H|\eta_{t-1})\eta(H)] \hat{\beta}_\eta^w + \hat{\beta}_R^w a_t^R + \hat{\beta}_S^w a_t^S, \\ E_t R_{t+1}^R &\equiv \hat{\beta}_0^R + [M(L|\eta_{t-1})\eta(L) + M(H|\eta_{t-1})\eta(H)] \hat{\beta}_\eta^R + \hat{\beta}_R^R a_t^R + \hat{\beta}_S^R a_t^S, \\ E_t R_{t+1}^S &\equiv \hat{\beta}_0^S + [M(L|\eta_{t-1})\eta(L) + M(H|\eta_{t-1})\eta(H)] \hat{\beta}_\eta^S + \hat{\beta}_R^S a_t^R + \hat{\beta}_S^S a_t^S. \end{aligned}$$

To be able to compute their forecast, period t households need to estimate η_t , which they do by using the Markov transition matrix M , and η_{t-1} . A further wrinkle to the problem has to do with simultaneity. To compute households estimates I need a_t^R and a_t^S , but these depend on $a_{2,t}$, $a_{3,t}$, $a_{4,t}$, and $a_{5,t}$, which are the variables I am trying to solve for, and for which I need the expectations in the first place. To solve this problem I set up a system of 4 equations (C.6)-(C.9) on 4 unknowns: $a_{2,t}$, $a_{3,t}$, $a_{4,t}$, and $a_{5,t}$. I can write the right-hand-side of these four equations as functions of past variables, and expectations which I can in turn write as functions of the unknowns I am looking for. For this last step, I need a Martingale assumption on aggregate asset types: $E_{t+k} a_{j,t+k} = a_{j,t}$ for $k = 1, 2, 3$. Note that this still allows me to take into account the direct (if not the indirect) effect of the demographic transition on future variables because when I aggregate cohort savings to obtain, for example, $E_{t+k} a_{t+k}^R$, I take into account the future evolution of cohort sizes, which households are assumed to know. Therefore, $E_{t+k} a_{t+k}^R = sh_{2,t+k} a_{2,t} + sh_{3,t+k} a_{3,t}$, for example. I also use this to compute higher order expectations for wages and interest rates like, for example, $E_t w_{t+2}$.

In steady-state, the solution that results from this expectation formation mechanism, like the rational expectations solution, has agents make zero average forecast mistakes. I illustrate this in

Figure 12 for wage expectations (the results are similar for interest rates expectations). Panel A shows the forecast errors for a particular simulation: in periods when the aggregate shock turns out to be high, agents tend to underpredict next-period's wages (dots above zero) and overpredict when the shock is low (dots below zero). Statistically I cannot reject, at the 5% level, the null that the mean forecast error is zero. Panel B shows wages and respective expectations averaged over multiple simulations – the invariant distribution means. Again, one can see that in the initial steady-state, expectations are very accurate, on average (basically the same point that panel A makes), but that some of this accuracy is lost in the transition period and is gradually recovered as time in the new steady-state increases. This is because agents take into account all the history they have available, so when the transition starts, most of their regressions sample pertains to the previous steady-state.

Figure 12: **Expectation formation performance**



E Algorithm

The algorithm proceeds in two large blocks. In the first block I solve for the equilibrium wage rate, w_t , and interest rates for the two assets: R_t^R and R_t^S . I do this by solving the producers' problem each period given current states: the aggregate shock, η_{t_1} , risky funds supplied, a_{t-1}^R , and safe funds supplied, a_{t-1}^S . Armed with prices, in a second block, I solve the households' problem and find saving decisions for next period $a_{2,t}$, $a_{3,t}$, $a_{4,t}$, and $a_{5,t}$, which allows me to redo the whole process for the next period. In more detail, here are the steps:

- (1) Supply initial asset holdings $a_{2,-1}$, $a_{3,-1}$, $a_{4,-1}$, and $a_{5,-1}$ (in practice this is an informed choice close to steady-state).
- (2) Start simulation with $\eta_0 = 1$ and draw a sequence of shocks $\{\eta_t\}_{t=1}^{S+T}$ using the Markov transition matrix M .

- (3) For every period $t = 0, \dots, S + T$, given states η_{t-1} , a_{t-1}^R , and a_{t-1}^S solve for labor market and asset markets clearing
 - (3.1) Interest rates loop: Guess interest rates $R_{0,t}^R, R_{0,t}^S$: If in first period provide informed guess, otherwise start with interest rates from previous periods
 - (3.1.1) Wage rate loop: guess a wage rate, $w_{0,t}$ and solve producers' problem to determine demand for labor
 - (3.1.2) Iterate on wage rate until labor market clears: if labor demand larger than labor supply (given by exogenous population evolution) adjust wage down; if larger adjust wage up; otherwise done
 - (3.2) Iterate on interest rates until both asset markets clear. This is a two-dimensional problem (unlike the labor market problem) that is more complicated and time consuming
 - (3.2.1) Given risky rate guess $R_{0,t}^R$ find a safe rate target $R_{T0,t}^S(R_{0,t}^R)$ that clears safe securities market
 - (3.2.2) Given safe rate guess $R_{0,t}^S$ find a risky rate target $R_{T0,t}^R(R_{0,t}^S)$ that clears risky securities market
 - (3.2.3) Check whether "guesses" and "targets" coincide. If so, done. If not, use an adjustment factor $\alpha \in (0, 1)$ and update guesses: $R_{1,t}^R = \alpha R_{0,t}^R + (1 - \alpha) R_{T0,t}^R$ and $R_{1,t}^S = \alpha R_{0,t}^S + (1 - \alpha) R_{T0,t}^S$ until convergence
- (4) Given states η_{t-1} , current savings $a_{2,t-1}, a_{3,t-1}, a_{4,t-1}, a_{5,t-1}$, and equilibrium prices w_t, R_t^R, R_t^S , solve the households' problem to find saving for next period $a_{2,t}, a_{3,t}, a_{4,t}, a_{5,t}$. See details in sections **C** and **D**.
- (5) Go back to (3) while $t \leq T + S$
- (6) Repeat steps (1) through (5) for N simulations, dropping the first S periods from each simulation. Take averages over the N simulations, resulting in an invariant distribution with T periods.

References

- ACEMOGLU, D., AND P. RESTREPO (2017): “Secular Stagnation? The Effect of Aging on Economic Growth in the Age of Automation,” *American Economic Review*, 107(5), 174–179.
- AKSOY, Y., H. S. BASSO, R. P. SMITH, AND T. GRASL (2019): “Demographic Structure and Macroeconomic Trends,” *American Economic Journal: Macroeconomics*, 11(1), 193–222.
- ALLEN, F., AND D. GALE (1988): “Optimal Security Design,” *The Review of Financial Studies*, 1(3), 229–63.
- AMARAL, P. S., AND E. QUINTIN (2020): “Security creation costs and economic development,” *Economic Theory*.
- AUCLERT, A., F. MARTENET, AND H. MALMBERG (2019): “Wealth and Demographics in the 21st Century,” 2019 Meeting Papers 952, Society for Economic Dynamics.
- BARRO, R. J., AND J. F. URSUA (2008): “Macroeconomic Crises since 1870,” *Brookings Papers on Economic Activity*, 39(1 (Spring), 255–350.
- BERNANKE, B. S., C. C. BERTAUT, L. P. DEMARCO, AND S. B. KAMIN (2011): “International capital flows and the returns to safe assets in the United States, 2003–2007,” International Finance Discussion Papers 1014, Board of Governors of the Federal Reserve System (U.S.).
- BIRDSALL, N., A. C. KELLEY, AND S. SINDING (eds.) (2003): *Population Matters: Demographic Change, Economic Growth, and Poverty in the Developing World*. Oxford University Press.
- BLOOM, D. E., D. CANNING, AND J. SEVILLA (2001): “Economic Growth and the Demographic Transition,” Working Paper 8685, National Bureau of Economic Research.
- BROCK, W. A., AND L. J. MIRMAN (1972): “Optimal economic growth and uncertainty: The discounted case,” *Journal of Economic Theory*, 4(3), 479–513.
- CABALLERO, R. J., E. FARHI, AND P.-O. GOURINCHAS (2017): “The Safe Assets Shortage Conundrum,” *Journal of Economic Perspectives*, 31(3), 29–46.
- CARVALHO, C., A. FERRERO, AND F. NECHIO (2016): “Demographics and real interest rates: Inspecting the mechanism,” *European Economic Review*, 88(C), 208–226.
- COALE, A., AND E. HOOVER (1958): *Population Growth and Economic Development in Low-Income Countries: A Case Study of India’s Prospects*. Princeton University Press, Princeton, NJ.
- CORBAE, D., AND E. QUINTIN (2016): “Asset Quality Dynamics,” 2016 Meeting Papers 418, Society for Economic Dynamics.
- DELVENTHAL, M., J. FERNÁNDEZ-VILLAYERDE, AND N. GUNNER (2019): “Demographic Transitions Across Time and Space,” Unpublished Manuscript.
- DJANKOV, S., R. L. PORTA, F. L. DE SILANES, AND A. SHLEIFER (2002): “The Regulation of Entry,” *The Quarterly Journal of Economics*, 117(1), 1–37.

- FAGERENG, A., C. GOTTLIEB, AND L. GUISO (2017): “Asset Market Participation and Portfolio Choice over the Life-Cycle,” *Journal of Finance*, 72(2), 705–750.
- FEYRER, J. (2007): “Demographics and Productivity,” *The Review of Economics and Statistics*, 89(1), 100–109.
- GAGNON, E., B. K. JOHANSEN, AND J. D. LOPEZ-SALIDO (2016): “Understanding the New Normal : The Role of Demographics,” Finance and Economics Discussion Series 2016-080, Board of Governors of the Federal Reserve System (U.S.).
- GALOR, O., AND D. WEIL (2000): “Population, Technology, and Growth: From Malthusian Stagnation to the Demographic Transition and Beyond,” *American Economic Review*, 90(4), 806–28.
- GUISO, L., M. HALIASSOS, AND T. JAPPELLI (2002): *Household Portfolios*. The MIT Press.
- IPARRAGUIRRE, J. L. (2020): *Ageing and Economic Growth and Development* pp. 397–527. Springer International Publishing, Cham.
- KARAHAN, F., B. PUGSLEY, AND A. ŞAHİN (2019): “Demographic Origins of the Startup Deficit,” NBER Working Papers 25874, National Bureau of Economic Research, Inc.
- LAGAKOS, D., B. MOLL, T. PORZIO, N. QIAN, AND T. SCHOELLMAN (2018): “Life Cycle Wage Growth across Countries,” *Journal of Political Economy*, 126(2), 797–849.
- LEE, R. (2016): “Macroeconomics, Aging, and Growth,” in *Handbook of the Economics of Population Aging*, ed. by J. Piggott, and A. Woodland, vol. 1 of *Handbook of the Economics of Population Aging*, chap. 0, pp. 59–118. Elsevier.
- LIANG, J., H. WANG, AND E. P. LAZEAR (2018): “Demographics and Entrepreneurship,” *Journal of Political Economy*, 126(S1), 140–196.
- LISACK, N., R. SAJEDI, AND G. THWAITES (2017): “Demographic trends and the real interest rate,” Bank of England working papers 701, Bank of England.
- LIU, W., AND P. POONPOLKUL (2020): “Demographic impacts on life cycle portfolios and financial market structures,” CAMA Working Papers 2020-20, Centre for Applied Macroeconomic Analysis, Crawford School of Public Policy, The Australian National University.
- LUNSFORD, K. G., AND K. D. WEST (2019): “Some Evidence on Secular Drivers of US Safe Real Rates,” *American Economic Journal: Macroeconomics*, 11(4), 113–139.
- MANCONI, A., E. NERETINA, AND L. RENNEBOOG (2018): “Underwriter Competition and Bargaining Power in the Corporate Bond Market,” Working Paper Series in Finance 548/2018, European Corporate Governance Institute.
- REITER, M. (2015): “Solving OLG Models with Many Cohorts, Asset Choice and Large Shocks,” Economics Series 320, Institute for Advanced Studies.
- RUPERT, P., AND G. ZANELLA (2015): “Revisiting wage, earnings, and hours profiles,” *Journal of Monetary Economics*, 72(C), 114–130.