

# A Threshold-Satisfying Competitive Location Model

Tammy Drezner and Zvi Drezner  
College of Business and Economics  
California State University  
Fullerton, CA 92834.

and

Shogo Shiode<sup>1</sup>  
Faculty of Economics  
Kobe Gakuin University  
Kobe, Japan

## Abstract

In this paper we consider a location model based on the threshold concept. We find the best location such that the probability of revenues falling short of the threshold is minimized. This objective is appropriate when a firm will not survive if its revenues fall below a known threshold.

A new store is to be located. Demand is not deterministic but rather has a statistical distribution. We seek the location at which the probability that the revenues (expressed as market share attracted by the new store) are below a given threshold is minimized. The model is formulated and solved, and computational results are given.

*Key words: threshold, competitive facility location, retailing.*

*JEL categorization: R30, R39.*

## 1. INTRODUCTION AND LITERATURE REVIEW

Common business objectives are to either minimize cost or maximize profit. When the cost is given, maximizing profit is equivalent to maximizing revenue. Under conditions of uncertainty, minimizing the variance of cost or profit is a desired objective because it reduces uncertainty. In this paper we propose the objective of minimizing the probability that revenues fall short of a given threshold necessary for survival. Such an objective is best to model the location decision of a new entrant to the market or any firm that is concerned about its survival. Normally, the main concern of a new entrant is

survival. Only after survival is secured, the objective may change to the more common one of maximizing profit.

This paper deals with stochastic optimization. Dantzig (1955) was the first to propose the concept of stochastic programming in the context of linear programming. Charnes and Cooper (1959) coined the term “chance constrained programming”. The first paper to introduce the threshold concept was by Kataoka (1963) in the context of transportation problems. He considered the minimization of the probability of falling below a pre-specified threshold. Frank (1966, 1967) considered a model of minimizing the probability that the cost function in the Weber or minimax problems (Love, Morris, and Wesolowsky, 1988) on a network exceeds a given threshold.

The concept of the threshold has been employed in financial circles as a form of insurance on a portfolio, either to protect the portfolio or to protect a firm’s minimum yield on investment (Jacobs and Levy, 1996; Olsen, 1997; Johansson et al., 1999; Finkelshtain et al., 1999).

The threshold concept is based on a requirement of meeting a certain minimum revenues, customers, market share, yield on investment, and so on necessary for survival and success of the company's operations and make a business venture worthwhile.

Recently, Serra et al. (1999) considered the problem of locating several facilities such that each facility attracts a minimum threshold of customers. In their model demand is deterministic and each facility *must* meet the threshold. Their model is an extension of the p-median model. Their analysis does not involve probabilistic issues. In this paper we focus on minimizing the *probability* of falling below the threshold rather than ascertaining that the minimum threshold is achieved.

The model is illustrated by applying the threshold concept to a location decision of a new retail outlet. In a competitive location context, the market share captured by the new store is a surrogate for revenue, hence the market share captured by the new store must meet or exceed a certain minimum threshold in order for the store to survive. The location objective, therefore, is to find the location that minimizes the probability of falling below the target revenue/market share threshold thus minimizing the chance that the new retail outlet will not survive.

## 2. THE NEW RETAIL FACILITY LOCATION PROBLEM

Consider a market area with a set of existing stores and customers who patronize these stores. A new store is planned to enter the market. Customers are aggregated in a set of communities. Buying power for each community, vital for modeling, is known for the present, but unknown for the future. We assume that the buying power at each community over the planning horizon is distributed according to some statistical distribution. This statistical distribution can be different for each community.

Customarily, when dealing with uncertain future market conditions, the objective is the maximization of the *expected* market share captured. In this paper we propose that there is a minimum market share threshold to be captured, below which the firm cannot survive. Therefore, the location objective becomes the minimization of the probability of falling short of the required threshold.

The future buying power at each community is drawn from a multivariate normal distribution. According to the central limit theorem, it is not essential for these distributions to be normal if there are more than 30 communities. It is likely that the distributions of buying power at two communities are positively correlated. This might be

due to good economic conditions or other factors resulting in either higher or lower than expected buying power in any community. If it is difficult to estimate the pair-wise correlation coefficients, it is recommended to use a common value for these correlation coefficients.

The following notation is used:

- $n$  the number of communities in the trading area,
- $k$  the number of existing facilities in the trading area,
- $B_i$  the buying power at community  $i$ ,
- $b_i$  the mean of the distribution of the buying power at community  $i$ ,
- $\sigma_i$  the standard deviation of the distribution of the buying power at community  $i$ ,
- $r_{im}$  the correlation coefficient between  $B_i$  and  $B_m$ .

For our analysis we use the gravity based model proposed by Huff (1964, 1966) and later refined and extended by Nakanishi and Cooper (1974), Fotheringham (1983), Fik and Mulligan (1990), Bell *et al.* (1998), and others. According to these models each facility has a known attractiveness level, and the probability that a customer selects a facility is proportional to its attractiveness and inversely proportional to some power of the distance to it. Consequently, a more attractive facility will attract customers from greater distances.

Additional notation:

- $d_{ij}$  the distance between community  $i$  and facility  $j$ ,<sup>2</sup>
- $A_j$  the attractiveness of facility  $j$ ,
- $A$  the attractiveness of the new facility,

$X = (x, y)$  the unknown location of the new facility,

$d_i(X)$  the distance between the new facility and community  $i$ ,

$\lambda$  the power to which the distance is raised (distance decay),

$T$  the threshold for the desired market share (given),

$M(X)$  the market share captured by the new facility at location  $X$ .

By gravity models the market share,  $M(X)$ , has the following distribution:

$$(1) \quad M(X) = \sum_{i=1}^n B_i \frac{A d_i^{-\lambda}(X)}{A d_i^{-\lambda}(X) + \sum_{j=1}^k A_j d_{ij}^{-\lambda}}$$

### 3. ANALYSIS OF THE THRESHOLD LOCATION PROBLEM

The distribution of the captured market share,  $M(X)$ , by equation (1) is a normal distribution for a given location  $X$ . The normality of the distribution is a result of the normality of the distribution of the  $B_i$ 's because other variables in (1) are constant for a given location  $X$ . If  $n$  is large enough (the commonly used cutoff is  $n$  being over 30), according to the central limit theorem the distribution of the market share  $M(X)$  is close to normality even when the  $B_i$ 's are not normally distributed.

The mean of the market share  $M(X)$  is:

$$(2) \quad \mu = \sum_{i=1}^n b_i \frac{A d_i^{-\lambda}(X)}{A d_i^{-\lambda}(X) + \sum_{j=1}^k A_j d_{ij}^{-\lambda}}$$

and the variance of the market share  $M(X)$  is:

$$(3) \quad \sigma^2 = \sum_{i=1}^n \sum_{m=1}^n r_{im} \sigma_i \sigma_m \frac{Ad_i^{-\lambda}(X)}{Ad_i^{-\lambda}(X) + \sum_{j=1}^k A_j d_{ij}^{-\lambda}} \times \frac{Ad_m^{-\lambda}(X)}{Ad_m^{-\lambda}(X) + \sum_{j=1}^k A_j d_{mj}^{-\lambda}}$$

The probability  $p(X)$  that the market share captured at the new store's location is below the threshold  $T$  is:  $p(X) = P(Z < \frac{T - \mu}{\sigma})$ . Since the cumulative normal distribution is

monotonically increasing, minimizing  $p(X)$  is equivalent to minimizing  $f(X) = \frac{T - \mu}{\sigma}$ .

Substituting (2) and (3) yields:

$$(4) \quad f(X) = \frac{T - \sum_{i=1}^n b_i \frac{Ad_i^{-\lambda}(X)}{Ad_i^{-\lambda}(X) + \sum_{j=1}^k A_j d_{ij}^{-\lambda}}}{\sqrt{\sum_{i=1}^n \sum_{m=1}^n r_{im} \sigma_i \sigma_m \frac{Ad_i^{-\lambda}(X)}{Ad_i^{-\lambda}(X) + \sum_{j=1}^k A_j d_{ij}^{-\lambda}} \times \frac{Ad_m^{-\lambda}(X)}{Ad_m^{-\lambda}(X) + \sum_{j=1}^k A_j d_{mj}^{-\lambda}}}}$$

The optimal location of the new store can be sensitive to the value of the threshold. There are three cases:

1. If the threshold is above the maximum expected market share, the probability of survival is lower than 50%. In this case a *greater* standard deviation is desirable because the probability of obtaining the threshold increases. This also follows the expression for  $f(X)$  (Equation (4)) in which the numerator ( $T - \mu$ ) is positive and thus the objective function is minimized for a larger denominator ( $\sigma$ ).
2. If the threshold is close to (either above or below) the maximum expected market share, the probability of survival is close to 50%, the objective of maximizing expected market share is dominant, and the value of the standard deviation has little

effect on the solution. In this case the numerator of  $f(X)$ ,  $(T-\mu)$ , is close to zero and thus the denominator ( $\sigma$ ) has little effect on the result.

3. If the threshold is below the maximum expected market share, the probability of survival is over 50%, the numerator  $(T-\mu)$  of  $f(X)$  is negative, and a lower standard deviation is desirable.

The function  $f(X)$  is not convex. Therefore, many local optima may exist. We propose to employ standard solution procedures, such as the solver in Excel, to solve the problem, using many, e.g. 100, different starting solutions. The iterative procedure can be done using a Visual Basic for Applications (VBA) macro. The macro calls the solver in Excel repeatedly (see Drezner and Wesolowsky, 2000, and the appendix in this paper for information on how to implement such a macro). The location, out of 100 experiments, that yields the minimum probability of failing to achieve the threshold, is selected as the solution to the problem.

#### 4. COMPUTATIONAL EXPERIMENTS

To illustrate the properties of the problem we tested this new approach on the test case presented in Drezner (1994, 1995). This test case consists of 100 communities arranged on a grid of 10 by 10 miles, and seven existing facilities. In Drezner (1994, 1995) the buying power of each community is 1. See Figure 1 for the depiction of the test problem.

Enter Figure 1 about here

For our test problems the buying power of each community is uniformly distributed with a mean of 1. Various values for the standard deviation of the buying power in each community were tested. We believe that buying power in different

communities is either unrelated or positively correlated. Therefore, we tested correlation coefficients of 0 and 0.1. We compare the probability of not meeting the threshold at the location found by our model with the probability of not meeting the threshold at the location found by standard models, namely, the location at which the captured market share is maximized.<sup>3</sup> The probability of not meeting the threshold at the location found by our model must be smaller than the probability of not meeting the threshold at the location at which the captured market share is maximized. The difference between these two probabilities is termed *the probability reduction* and is denoted by  $\Delta p(T)$ . The probability reduction depends on the threshold.

In order to test the sensitivity of the location solution to the value of the threshold  $T$ , we solved the problem for various threshold levels, 21 thresholds for each test problem. The list of thresholds tested were evenly spaced and ranged from  $\text{Min}\{T\}$  to  $\text{Max}\{T\}$ , and a given increment. Since the maximum possible captured market share is 12.96 (Drezner et al., 2000), we centered the list of thresholds at 13. The parameters for the experiments are summarized in Table 1.

Insert Table 1 around here
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We observed the effect of the threshold on *both* the optimal location and the probability of survival. Figure 2 depicts the trajectory of the optimal location as the threshold moves from  $\text{Min}\{T\}$  to  $\text{Max}\{T\}$ . For all four test problems, the optimal location moves from left to right with a discontinuous trajectory. The discontinuity occurs for a threshold below the maximum expected market share (12.96). It follows that if the threshold is appreciably below the maximum possible captured market share (or

alternatively, when the probability of survival is high), the best location is in one region of the market area. When the probability of survival is low, the best location is close to the location at which the market share is maximized. This result is as expected. When the probability of survival is high, the firm should focus on minimizing the standard deviation that occurs at a different region of the market area. However, when the probability of survival is low, the firm should strive to maximize its expected market share, and essentially disregard the standard deviation.

The results for test problem #1 are also reported in Table 2. Table 2 depicts the following information about test problem #1:

Column 1: The threshold  $T$  given in 0.2 increments for the sensitivity analysis;

Column 2: The  $(x,y)$  location coordinates at which the probability of failure for the given threshold (column 1) is minimized;

Column 3: The probability of failure to meet the threshold at the location in column 2:

Column 4: The probability of failure to meet the threshold at the location that maximizes the expected market share  $\mu$  by Equation (2) ((6.43, 5.55) according to Drezner et al., 2000) which is the location that is recommended by existing methods. The difference between Column 3 and Column 4 is the probability reduction  $\Delta p(T)$ .

Insert Table 2 around here
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As can be seen from Table 2, the probability of failure to meet the threshold is cut by about one half for small probabilities.

In Figure 3 we depict the probability reduction  $\Delta p(T)$ . It is interesting to observe that the probability reduction function has a bi-modal distribution. The most reduction in probability occurs for a threshold below the maximum expected market share (with the probability of failure to meet the threshold around 5-10%), with a secondary mode for a threshold greater than the maximum expected market share (with probability of failure to meet the threshold around 90%). When the threshold is equal to the maximum possible market share, there is no reduction in the probability. Both probabilities are exactly 50%.

Insert Figures 2 and 3 about here

## 5. THE SIMULATIONS

In order to validate the probabilities of failure to meet the threshold calculated by the model, we ran a series of simulations, a simulation for each threshold for each test problem. In the simulation, the following was repeated 10,000 times:

- 1) The buying power values for the 100 communities were drawn from a multivariate distribution as follows:
  - a) One hundred random values  $R_i$  were drawn from a uniform distribution in the segment  $[-0.5, 0.5]$ .
  - b) The sum,  $S$ , of all drawn  $R$ 's was calculated.
  - c) The individual buying power  $B_i$  was calculated as  $1 + eS + fR_i$  for appropriate  $e$  and  $f$  such that the mean is 1, the standard deviation is  $\sigma$ , and the correlation coefficient between  $B_i$  and  $B_m$  is  $r$ . Since the variance of a uniform distribution over a segment of length 1 is  $1/12$ ,  $Var(R_i) = 1/12$ ,  $e$  and  $f$  are the solution to the equations (for  $n=100$  communities):

$$(n-1)e^2 + (e+f)^2 = 12\sigma^2$$

$$(n-2)e^2 + 2e(e+f) = 12r\sigma^2$$

which leads to  $f^2 = 12(1-r)\sigma^2$ ;  $e = f \frac{\sqrt{1 + \frac{nr}{1-r}} - 1}{n}$ .

- 2) The market share captured by the new facility at the location found by the procedure is calculated.
- 3) The number of cases (out of 10,000) for which the threshold  $T$  is not met is found.

Insert Table 3 around here

The results of the simulations for all problems are depicted in Table 3. The last row in the table is the sum of squares of the standardized differences between the expected and simulated values. For a calculated probability of failure  $p$ , the expected number of instances which do not meet the threshold is  $10,000p$  (the rounded value is reported in the table) with standard deviation of  $100\sqrt{p(1-p)}$ . The sum of squares of the standardized scores approximates a  $\chi^2$  distribution with 20 degrees of freedom. The mean of a  $\chi^2$  distribution with 20 degrees of freedom is equal to 20. The sum of squares reported in the table yield p-values between 0.36 and 0.55, meaning that the simulated results are not significantly different from the expected results.

## 6. DISCUSSION

In this paper we discuss the location of a new retail facility. We propose a different objective function by observing that there is a market share threshold to be captured, below which a firm will not survive. The appropriate objective function is to

minimize the probability that the firm will not achieve the threshold and thus will not survive. It follows, therefore, that rather than locating a facility to maximize market share, a firm should locate so as to minimize the *probability* of falling short of this minimum threshold.

We solve the problem in the plane using the gravity model. The models can be easily adapted to other competitive location models and to different environments such as location on a network.

The optimal location tends to be different from the location at which the expected market share is maximized. For our test problems we found that the location is different especially when the probability of failure is relatively small. When the probability of failure is high (50% or more), the company should concentrate on maximizing the expected market share and locate close to the location at which the market share is maximized. When the probability of failure is low, the firm should concentrate on minimizing the variance thus reducing the uncertainty. This may yield a location in a different region of the market area.

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## Appendix

A spreadsheet that calculates the objective function (4) for given values of  $x$  and  $y$  is required. We prepared a spreadsheet where the objective function (4) is programmed in cell [L3] with the variables  $x$  in cell [K1] and  $y$  in cell [K2]. The Solver parameters are established. When the Visual Basic for Applications (VBA) macro window is open, click “Tools”, then “References” and make sure that Solver.xls is ticked.

The “heart” of the VBA program may look like this:

```
' 21 different thresholds entered in cell [B1], one at a time, with an
' average threshold of 13 and an increment given in cell [A4].

For it = 1 To 21
[B1] = 13 + [A4] * (it - 1)

' Solving the problem 100 times from random starting locations, and
' recording the best solution in fmin.

fmin = 1000000#
For i = 1 To 100
[K1] = Rnd() * 10 ' Generating a random starting point
[K2] = Rnd() * 10
For iter = 1 To 10 ' Calling Solver up to 10 times
indx = solversolve(userfinish:="true") ' Calling Solver
' indx=0 means that there was no error message, i.e., solution found.
x = solverfinish(keepfinal:="1") ' Updating x and y
If (indx = 0) Then GoTo out ' exit loop if there is no error return
Next iter
out:
If [L3] < fmin Then
fmin = [L3]
xmin = [K1]
ymin = [K2]
End If
Next i

' Recording the best result for the threshold.

Cells(it + 1, 14) = [B1] ' The Cells command refers to
Cells(it + 1, 15) = xmin ' specific cells in the spreadsheet.
Cells(it + 1, 16) = ymin ' Cells(i,j) refers to the cell in
Cells(it + 1, 17) = fmin ' row #i and column #j.
' Calculating the probability:
Cells(it + 1, 18) = WorksheetFunction.NormSDist(Cells(it + 1, 17))

' Calculating the probability at the location which maximizes the
' expected market share.

[K1] = 6.43
[K2] = 5.55
Cells(it + 1, 19) = [L3]
Cells(it + 1, 20) = WorksheetFunction.NormSDist(Cells(it + 1, 19))
Next it
```

Table 1: Parameters for the test problems

Test Problem	r	$\sigma$	Threshold Range		Increment
			Min	Max	
1	0	0.25	11.0	15.0	0.2
2	0	0.3	10.0	16.0	0.3
3	0.1	0.25	9.0	17.0	0.4
4	0.1	0.3	8.0	18.0	0.5

Table 2: Results for the test case of the model and simulations for various thresholds.

(1)	(2)		(3)	(4)
Threshold	Best Location		Model's Objective Probability	*
	<i>x</i>	<i>y</i>		
11.0	5.59	5.38	$6.58 \times 10^{-6}$	$1.92 \times 10^{-5}$
11.2	5.62	5.42	$4.7 \times 10^{-5}$	0.00011
11.4	5.64	5.45	0.00028	0.00052
11.6	5.67	5.47	0.00132	0.00212
11.8	5.69	5.49	0.00525	0.00733
12.0	5.72	5.51	0.01733	0.02163
12.2	5.74	5.53	0.04776	0.05458
12.4	5.77	5.54	0.11072	0.11845
12.6	5.81	5.56	0.21782	0.22258
12.8	6.37	5.55	0.36500	0.36532
13.0	6.44	5.55	0.52985	0.52987
13.2	6.49	5.54	0.68887	0.68941
13.4	6.53	5.54	0.81827	0.81949
13.6	6.57	5.54	0.90719	0.90868
13.8	6.60	5.54	0.95885	0.96010
14.0	6.62	5.54	0.98424	0.98504
14.2	6.65	5.53	0.99480	0.99521
14.4	6.67	5.53	0.99853	0.99869
14.6	6.69	5.53	0.99964	0.99970
14.8	6.72	5.53	0.99993	0.99994
15.0	6.74	5.53	0.99999	0.99999

\* The probability of failure to meet the threshold at the location which maximizes the expected market share.

Table3: Simulation results of 10,000 cases

r=0; $\sigma=0.25$			r=0; $\sigma=0.3$			r=0.1; $\sigma=0.25$			r=0.1; $\sigma=0.3$		
T	Exp.	Sim.	T	Exp.	Sim.	T	Exp.	Sim.	T	Exp.	Sim.
11.0	0	0	10.0	0	0	9.0	2	1	8.0	1	0
11.2	0	0	10.3	0	0	9.4	7	8	8.5	4	5
11.4	3	0	10.6	0	0	9.8	23	16	9.0	15	11
11.6	13	7	10.9	1	0	10.2	66	49	9.5	48	36
11.8	53	41	11.2	6	4	10.6	170	140	10.0	133	108
12.0	173	166	11.5	35	35	11.0	393	380	10.5	328	310
12.2	478	452	11.8	165	185	11.4	810	805	11.0	714	713
12.4	1107	1125	12.1	577	557	11.8	1492	1447	11.5	1378	1328
12.6	2178	2193	12.4	1541	1615	12.2	2475	2455	12.0	2365	2328
12.8	3650	3669	12.7	3208	3263	12.6	3725	3763	12.5	3649	3681
13.0	5298	5309	13.0	5249	5255	13.0	5127	5115	13.0	5106	5094
13.2	6889	6876	13.3	7204	7194	13.4	6513	6517	13.5	6548	6558
13.4	8183	8137	13.6	8650	8603	13.8	7721	7700	14.0	7794	7780
13.6	9072	9021	13.9	9474	9456	14.2	8649	8640	14.5	8732	8720
13.8	9588	9582	14.2	9836	9848	14.6	9277	9296	15.0	9349	9375
14.0	9842	9865	14.5	9960	9961	15.0	9653	9629	15.5	9703	9687
14.2	9948	9952	14.8	9992	9996	15.4	9851	9848	16.0	9880	9873
14.4	9985	9986	15.1	9999	10000	15.8	9943	9936	16.5	9957	9952
14.6	9996	9998	15.4	10000	9999	16.2	9980	9983	17.0	9986	9987
14.8	9999	10000	15.7	10000	10000	16.6	9994	9994	17.5	9996	9996
15.0	10000	10000	16.0	10000	10000	17.0	9998	10000	18.0	9999	10000
SS=21.02			SS=21.70			SS=20.71			SS=18.58		

Figure 1: The Test Problem

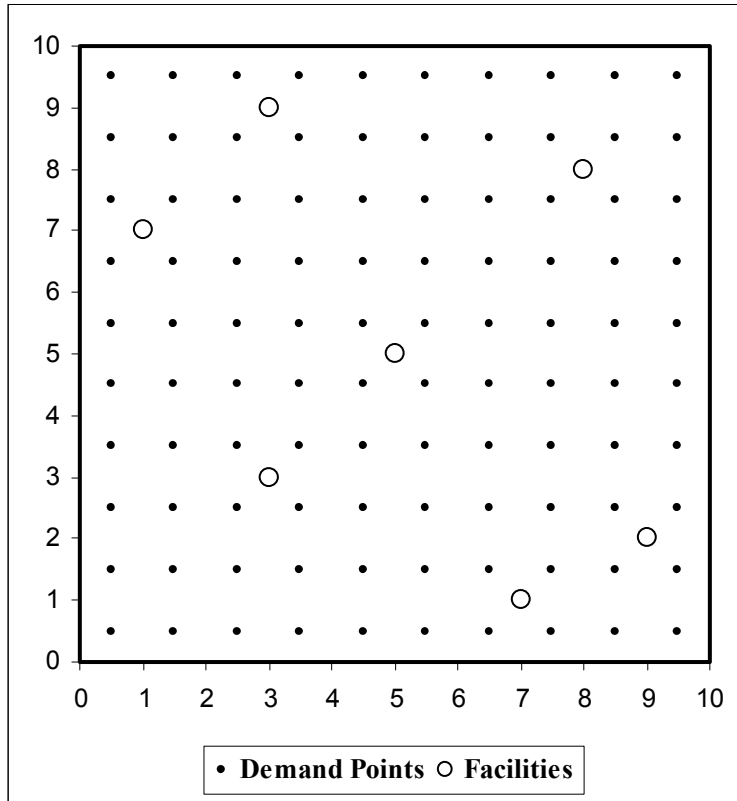
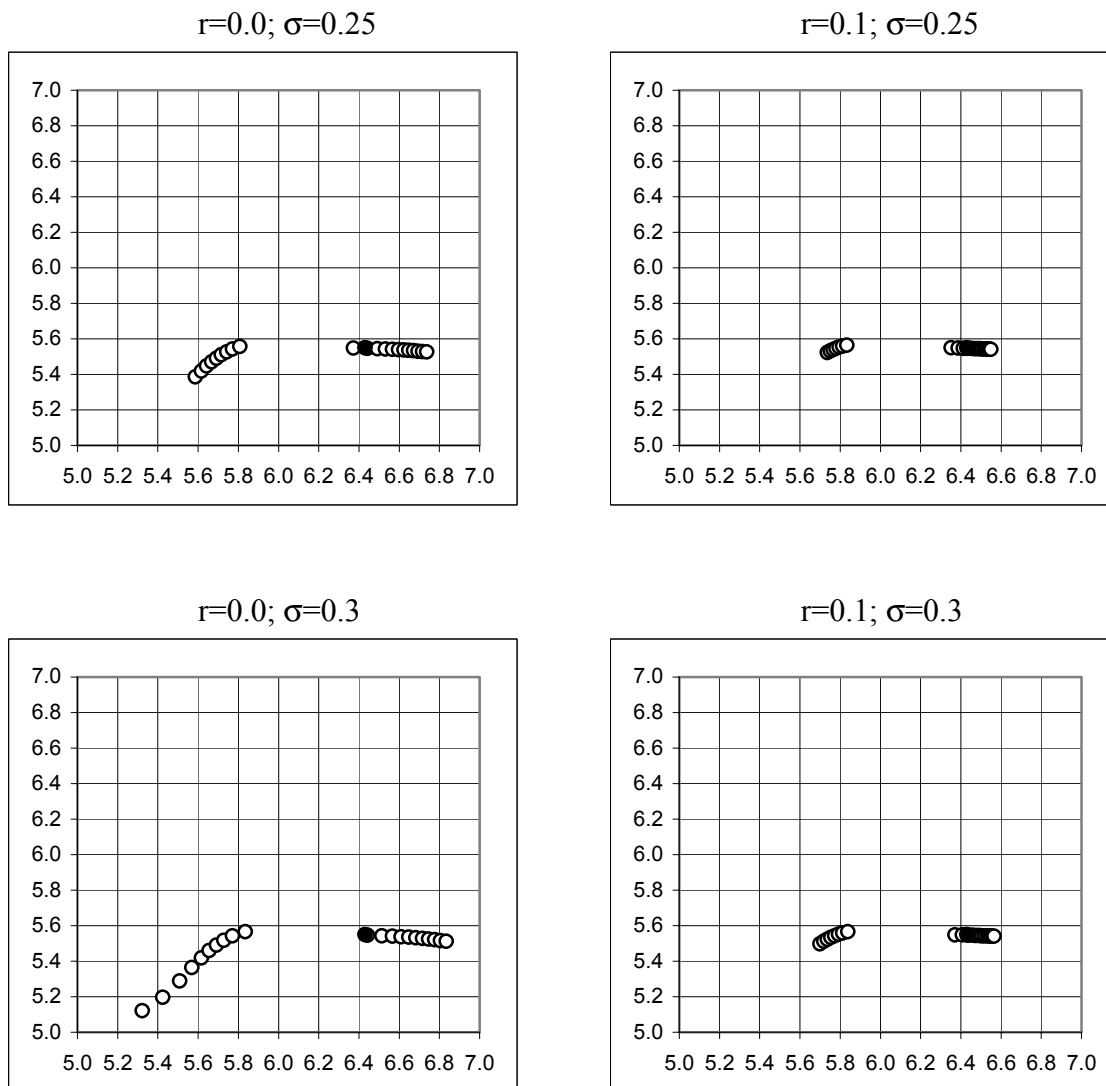


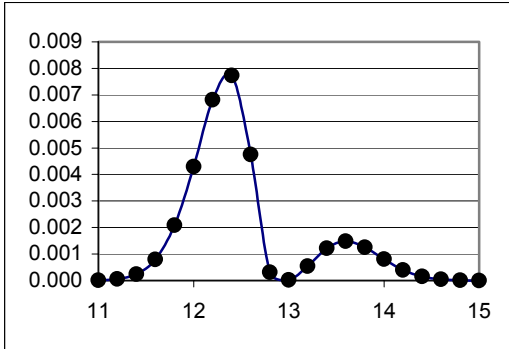
Figure 2: Best Locations



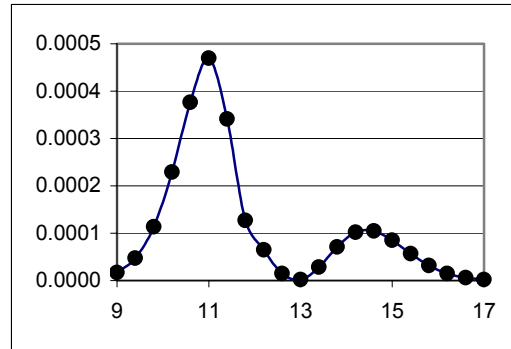
- Location at which the expected market share is maximized (it is the same location for all test problems);
- Location at which the probability of failure is minimized for a given threshold.

Figure 3: Probability Reduction  $\Delta p(T)$

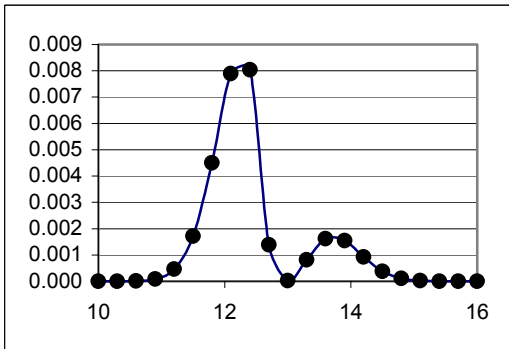
$r = 0.0; \sigma = 0.25$



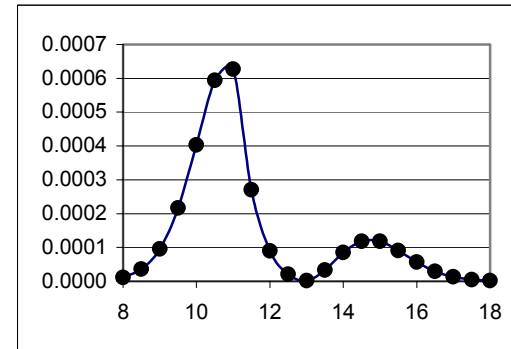
$r = 0.1; \sigma = 0.25$



$r = 0.0; \sigma = 0.3$



$r = 0.1; \sigma = 0.3$



## List of Figures

Figure 1: The Test Problem

Figure 2: Best Locations

Figure 3: Probability Reduction  $\Delta p(T)$

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<sup>1</sup> Part of this research was done while the third author was visiting California State University-Fullerton.

<sup>2</sup> The methodology employed in this paper can be used for any expression of the distance. In our computational experiments we opted to use Euclidean distances corrected for the areas of the communities as explained in Drezner and Drezner (1997). This distance correction accounts for the bias introduced to the distance customers travel to a facility. Patrons residing in the same community have varying travel distances, because the community is not a mathematical point. A distance correction of  $\alpha$  means replacing the distance  $d$  with  $\sqrt{d^2 + \alpha}$ .

<sup>3</sup> Maximizing the expected market share captured is identical to the problem solved in Drezner (1994, 1995) and Drezner *et al.* (2000) because the standard deviations and correlation coefficients introduced in the current paper do not affect the *expected* captured market share. Using a distance correction of 0.24 (Drezner and Drezner, 1977), the location which maximizes the expected market share captured is at (6.43,5.55) with a market share of 12.96 (Drezner et al., 2000).